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Estimation of product category sales responsiveness to allocated shelf space

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Abstract

A retail chain manager must draw on experience based on data available from his points of sale to diagnose space misallocations in stores and to make recommendations. This paper presents an empirical estimate of shelf space elasticities from a variety store chain database at product category level with a share of space vs. share of sales econometric model. It suggests that external influences could explain space elasticity differences. Results show that space elasticities increase with the impulse buying rate of the product category and do not depend on the type of store.

Author Keywords: Space allocation; Shelf space elasticities; Impulse buying

Article Outline

1. Introduction

Traditional retailing formats for consumer goods have entered the mature phase of their life cycle. After a euphoric period of internal growth, there has been a vast movement of external growth; the retailing industry is thus highly concentrated. Competition is intense both among stores with the same retailing format (intra-format competition) and among stores with different formats (inter-format competition). Under these highly competitive conditions, retailers have two concerns: on the one hand, they are looking for differentiation to avoid price competition and to acquire a long term advantage in the consumer's mind; on the other hand, they are seeking productivity gains (by cost reductions or economies of scale) and the integration of decisions to share experience.

Centralizing such functions as purchasing, logistics or merchandising at a higher level than that of the store is one means of reaching that double goal and appears to be an effective way of achieving a homogeneous set of points of sale that reflect the market position of the retail chain. Moreover, decision centralization is a way to rationalize and reduce the cost of these functions (Sinigaglia et al., 1995). Our research focuses on the control exerted by a store chain on a major merchandising decision: the allocation of space among product categories within the set of stores. Actually, selling space is a scarce and critical resource which must be allocated so as to optimize the profitability level of the assortment, and retail chains need methods for validating the pertinence of the shelf space allocation made by store managers.

There are two complementary but radically different hierarchical approaches to managing space allocation (Fig. 1). The bottom–up approach consists of allocating space to each Stock Keeping Unit (SKU) on the basis of handling costs and visibility constraints (Bultez and Naert, 1988; Bultez et al., 1989 and Bultez et al., 1995) and determining (by a process of aggregation and negotiation) each sub-product category's and product category's shelf space. This space allocation process is guided by operational concerns: a store manager who wants to

implement his store thinks in these terms. By contrast, the top–down approach, on which we focus, leads to computing norms (from collected space, sales and profit data) that permit a local diagnostic of space allocation performance in each store. This benchmarking process is exerted by the retail chain from more to less aggregate levels of the product classification. Although different, these approaches are complementary. In the past, space allocation research was based largely on a bottom–up approach and those models cannot, therefore, be applied directly to our field of research.

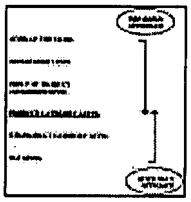 **Display Full Size version of this image** (6K)

Fig. 1. Hierarchical approaches to managing space allocation.

Currently, the norm-based diagnostics system used by retailers does not take account of the fact that responsiveness to space allocation varies across product categories and stores. In order to test the validity of this simplifying assumption, we propose an empirical estimate of shelf space elasticities from a variety store chain database at the product category level, and suggest some external influences that could explain differences in space elasticity.

Section 2 of this paper presents a conceptual framework based on practice and on the literature in order to understand the link between sales and space allocated to product categories, and to highlight the parameters that could moderate this relationship. Section 3 presents an econometrically estimated share of store sales vs. share of store space relationship. Empirical results on space elasticities are then presented and discussed.

2. Allocating space between product categories

2.1. The managerial context

Retailers' practice reveals that shelf space, together with factors such as price, assortment or promotions, contribute to increased sales, with a tendency towards decreasing returns. But the causal link can be reversed because retailers anticipate demand peaks for some highly seasonal product categories by increasing their share of space: in this case, anticipated sales drive space rather than the reverse. However, selling space is seen as a limited resource and retailers must ensure periodically that it is correctly used in the stores.

The development of category management has led retailers, manufacturers and researchers to work on space allocation problems within product categories (with a bottom–up approach) rather than at aggregate levels, which is the perspective of retail chains (top–down approach). In particular, prompted by manufacturers who are trying to improve the display of their brands, commercial programs focusing on supply aspects (such as Spaceman or Apollo) have been developed to facilitate shelf space layout. Knowing the physical characteristics of the products, these programs allocate shelf space according to traditional criteria such as turnover *pro rata*, gross profit or margin *pro rata*, under constraints such as handling or inventory costs. Despite their graphical and empirical advantages, these programs consider the relationship between sales and shelf space to be linear and constant across products, categories and stores, which is a weakness. They cannot, therefore, be seen as

effective shelf space optimization tools within a product category, and are even less able to help a retail chain manager who wants to enhance the global space productivity of each point of sale.

Retailers know that sales–space elasticities vary with product category, store characteristics and with the environment in which the store is located. They do not take these influences into account in their diagnostic process but rather use store typologies, in which points of sale have quite homogeneous physical and environmental characteristics, compute efficiency norms, and diagnose whether the shelf space allocated to a product category in a store is too small, satisfactory, or too large. They cannot, however, quantify precisely the consequences of shelf space modifications on the store's sales and profitability primarily because they do not use estimates of shelf space elasticities.

2.2. The sales–space relationship

The literature on visual perception supports the view that space has a positive influence on sales for a given product or group of products. Consumers walking around a store have a flattened cone of peripheral vision through which they automatically and subconsciously scan the merchandise they pass (Phillips and Bradshaw, 1993). Therefore, if a product is given a large shelf space, it is likely to be visually perceived, sifted out and bought more frequently (owing to the bulk-effect created). The literature considering shelf space as stocking space (Borin et al., 1994, for example) also provides evidence for the existence of a causal effect of space allocated on sales. If more space is allocated to a product category, the SKUs of this category will be out of stock less often, and the total sales of the category will increase.

In the literature, it is usually assumed that the marginal returns from allocated shelf space are regressive: as there is a fixed market saturation level, each additional space unit allocated to a product or a product category yields less than the previous one. This hypothesis results in the choice of a multiplicative power function with space elasticities lower than 1 in the models of Corstjens and Doyle (1981) and Corstjens and Doyle (1983) and Borin et al. (1994). When the actual allocated space is far from optimal or even nonexistent, marginal returns are supposed first to increase and then to decrease in an S curve (Bultez and Naert, 1988; Bultez et al., 1989; Drèze et al., 1994).

The diversity in empirical estimates of space elasticities (most vary between 0.15 and 0.8) is mainly due to the methodology and data used and to the level of the nomenclature studied. Among many studies using store experimentation (Cox, 1964; Kotzan and Evanson, 1969; Cox, 1970; Curhan, 1972 and Curhan, 1973; Drèze et al., 1994), the most interesting and reliable are the study of Curhan (1972) which, taking a large sample, found an average value of 0.212 for space elasticity, and the large scale work of Drèze et al. (1994) which concluded that the number of facings allocated to a product is one of the least important success factors (whereas the position of the product on the shelf seems to be far more important for determining sales). Both draw the conclusion that space elasticity weakness is due to the fact that most products receive an over-allocation of shelf space. Referring to 20 experiments, Heinsbroek (1977) also noticed a low average space elasticity (0.15) at the item level, with elasticities below 0.05 in 40% of the cases and none exceeding 0.5. Other researchers have estimated space elasticities using cross-sectional data. Thus, in their research, Corstjens and Doyle (1981) found a low mean value of 0.086. At store level, Thurik (1988) found a much higher average space elasticity: close to 0.6 for department stores, supermarkets and hypermarkets. For mail order selling, Desmet (1991) also found a high space elasticity (0.812).

Another field of research, inaugurated by Anderson and Amato (1974), Hansen and Heinsbroek (1979) and Anderson (1979), has provided space allocation optimization models. Corstjens and Doyle (1981) and Corstjens

and Doyle (1983) highlighted the role of sales–space cross-elasticities (which measure the sales responsiveness of one product to the space allocated to other products) and, by taking these into account, developed the first space allocation model incorporating interactions. They found that for many of the product categories, cross-elasticities are lower than direct elasticities but statistically significant. They concluded that the common approach of ignoring them leads to significantly sub-optimal space allocation. Bultez and Naert (1988), Bultez et al. (1989) (SHARP models), Borin et al. (1994) and Borin and Farris (1995) continued in this direction by modeling interactions between items of a product category through cross-elasticities and symmetric or asymmetric attraction models.

Space allocation models for items within a product category, such as SHARP models, cannot be readily applied in their original form to product categories because they are based on the assumption that demand interdependencies result from cannibalism (products are considered substitutes) within each product category, entirely independent from the others. In our case, the links between the demand for product categories are more complex: cross-elasticities can be either negative because both the size of the household's basket (literally and figuratively) and shopping time are limited, or positive (Betancourt and Gautschi, 1990) as a product category can be 'traffic-building', attracting customers to the store and creating additional demand for other product categories.

Like Corstjens and Doyle (1981), we consider that the sales–space relationship is a multiplicative power function with a saturation effect. But, unlike them, we will not take cross-elasticities into account, for several reasons. Firstly, Corstjens and Doyle established that direct space elasticities are higher than cross-elasticities and we consider that the emphasis must first be placed on measuring the impact of the greatest effects. Secondly, we assume that the significant values for cross-elasticities they found are partly due to the type of stores in which data were collected: a chain of stores that sold only five categories of products (chocolates, toffees, candies, greeting cards and ice creams) that can easily substitute for or complement one another. This does not seem to be the case in supermarkets at a more aggregate level, and therefore cross-elasticities are likely to be weaker and less crucial to study. Moreover, in supermarkets, the number of product categories is higher: it is very difficult to obtain reliable estimates for cross-elasticities. We therefore focus on direct space elasticities and test the influence of store and category parameters which modify the sales–space relationship.

The direction of causality is the primary problem that researchers have to face with regard to the sales–space relationship. Theoretically, experimentation is the only way to avoid this issue. But this methodology has its shortcomings: in-store experimentation is very expensive, difficult to implement on a large scale, and often rather inconclusive (Corstjens and Doyle, 1981). Many studies, therefore, use cross-sectional data. This methodology can be used to detect a correlation between differences in space allocation and variations of demand but does not provide proof of the existence of a causal link between the variables. Frank and Massy (1970) first discussed the issue of causality, giving three main hypotheses that might explain the positive relationship between sales and space with time-series and cross-sectional data: space has a positive influence on sales (conventional hypothesis); space is the store manager's response to anticipated sales (the causality link is reversed); and the effect of shelf space is directly linked to differences in store size. They concluded that it would be ideal to conduct observational and controlled store experiments on the same set of stores to validate the results of observational studies. In sum, the causality issue is very critical and the results of cross-sectional observations must be carefully treated. Our methodology is subject to these constraints.

2.3. Conceptual framework

Authors such as Curhan (1972) have put forward that the relationship between shelf space and unit sales is not

uniform and have tried to emphasize that some product attributes and store attributes can influence the sales–space relationship. The high space elasticity variance across experimental studies seems to support this hypothesis. Fig. 2 presents our conceptual framework with the direct relationship between space and sales that is to be estimated and the moderating variables affecting it.

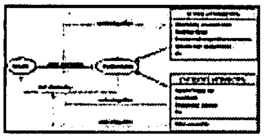 Display Full Size version of this image (14K)

Fig. 2. Conceptual framework.

2.3.1. The moderating role of store attributes

Elaborating on the view of Frank and Massy (1970) who postulate that shelf space elasticities vary with store size, we propose that the following store attributes can also influence the sales–space relationship.

2.3.1.1. Membership in a store chain and retailing format

A store chain provides a certain type of supply, consistent with the image that it wishes to project through its points of sale. The store chain image can influence the sales of some product categories and therefore affect their space elasticities. The store's clientele and supply also depend on its retailing format which is likely to affect the sensitivity of sales to space. For example, shelf space elasticities should be higher for self-service stores than for stores with salesmen.

2.3.1.2. Location and competitive environment

Space elasticity variations could be explained by the economic potential (population and wealth) and by the competition intensity of the stores' trade areas. Although these variables have been ignored in the literature, it seems reasonable that a store located in an area characterized by weak purchasing power will have lower space elasticities than a store in a residential area, because households are likely to focus on price rather than on presentation parameters. If we take another example, this time of the do-it-yourself department of a variety store, this product category is likely to be greatly space-unresponsive if there is a proximate category killer specialized in do-it-yourself items, because no matter what the variety store attempts to do, the competitor's supply remains more attractive. In the first example, the general level of space elasticities is concerned by the environment of the store, whereas in the second, only a single elasticity is affected.

2.3.1.3. Specific store characteristics

Among a range of store characteristics, store size is likely to be one of the most crucial, and in any case the only attribute whose influence on space elasticities has been discussed in the literature. As marginal returns from allocated shelf space are supposed to be regressive, the larger the category (and, therefore, the larger the store), the weaker space elasticities should be. However, large stores attract many customers who patronize several points of sale and are more sensitive to choice and impulse buying: they provide a higher potential demand per square meter than in small stores, which can lead to higher space elasticities. The literature gives contradictory results: at the product level, Frank and Massy (1970) find that the turnover is more sensitive to

space allocation increases in high volume stores than in low volume stores. At the store level, however, Thurik (1988) establishes that space elasticity decreases with store size: space elasticity amounts to 0.68 for supermarkets and to 0.51 for hypermarkets. Empirical results for sales–promotion elasticity also support the decreasing elasticities hypothesis: 2.07 for supermarkets, 1.94 for hypermarkets and 1.87 for large hypermarkets (LSA, 1995). Other specific store characteristics may also have an influence on the level of space elasticities. For example, the presence of a car park in a store might make people more likely to buy products, especially large and bulky items, under the influence of the conditions of their presentation.

2.3.2. The moderating role of product category attributes

Sales–space elasticity is also expected to vary according to the group of products studied and the buying behavior of customers with respect to them.

2.3.2.1. Impulse buying rate of the category

Impulse buying is a well-known and widespread in-store behavior but a very controversial concept. The major divergence point in this research field is linked to the fact that impulse buying can be studied as a product-specific phenomenon or as a consumer characteristic. It results in both an operational approach to the concept (on which we focus), whose objective is to quantify a product phenomenon with a view to be used by retailers, and a behavioral approach, aimed at better understanding the consumer (Leblanc-Maridor, 1989). The first approach defines impulse buying as a function of the place where the buying decision was made (in-store decision-making). By contrast, the behavioral definition is based on consumer characteristics: the speed of the decision making, the small amount of information necessary to buy the product, emotional state... The operationalization of impulse buying as a product characteristic varied considerably, resulting in findings that are not always consistent across studies (Cobb and Hoyer, 1986). Impulse buying has been measured mainly by the difference between purchase outcomes and intentions collected before entering the store (Du Pont de Nemours and Compagny, 1965; Point-of-Purchase Advertising Institute, 1963), and by a posteriori questionnaires (Bellenger et al. (1978) asked, for each item bought, 'when did you decide you wanted to buy this product?'). In both cases, (beyond the wide variations in time frame and stores studied), biases differ and therefore different phenomena are measured. Nevertheless, despite their weaknesses, these studies provide useful classifications of the impulse buying rate of products. We chose to use Bellenger et al. (1978) results because data are available for a large number of product categories that are present in the assortment of the retail chain we study. The authors highlighted costume jewelry, ladies' shoes, curtains and draperies, bakery products, meals and snacks, women's wear, books and stationery as being characterized by a high level of impulse buying. As far as the link between impulse buying and space elasticities is concerned, Brown and Tucker (1961) first assumed that impulse buying would increase the importance of exposure parameters, such as shelf space, relative to other product parameters. They suggested that sales–space elasticity rises across three classes of products: *unresponsive products* (commodities such as salt or sugar), *general use products* (staples such as breakfast food or canned fruits) and *occasional purchase products* (impulse buying such as candy or nuts). Cox (1964) and Cox (1970) and Curhan (1972) found impulse products to be more space elastic than staples. We try to provide support for this assumption by comparing highly space-elastic product categories with product categories characterized by a high level of impulse buying (using the typology defined by Bellenger et al. (1978) and the practice of marketing managers in retail firms). We must observe the importance of a double source of information concerning the level of impulsiveness of product categories. It would have been dangerous to limit ourselves to managers' know-how to define highly impulsive product categories as they may unconsciously associate impulse buying categories with highly space-elastic ones and therefore define highly impulsive categories by highly space-sensitive ones. In this case, the positive relationship between impulse buying and

high space elasticities would result from a tautology. Therefore, despite its shortcomings, we use a classification based on consumers, as a safety precaution.

2.3.2.2. Assortment

Space allocation decisions across and within product categories cannot be considered to be independent. Increasing allocated space to a category can cover different realities in terms of assortment choices, and lead to different levels of sales responsiveness. If the additional space is distributed between the same SKUs, then, if products were frequently out of stock, sales would probably rise sharply. If, however, this were not the case, increased visibility rather than increased availability, would help augment sales, probably with decreasing returns. Alternatively, if new items were added to the selection, the new SKUs could be direct substitutes for the original SKUs and the sales of the category would level off while the turnover per square meter would drop. On the other hand, new items can create a complementary demand leading to increased sales. In this case, the turnover per square meter will not increase unless the new items are more profitable than the average of the original items. We nonetheless assume that the section manager looks for maximum profitability from shelves and should therefore make the best use of additional space that may be allocated to product categories. SHARP models can provide useful recommendations for effective intra-category space allocation decisions. Therefore, our approach applies to the retail chain, is limited to an aggregate level of product classification, and does not consider assortment issues.

2.3.2.3. Promotional intensity

Promotions have a powerful influence on the sales and space allocated at SKU level. They probably influence the sensitivity of sales to space, particularly when the promotion is a display. The impact, however, is likely to be weaker at category level. Moreover, the monthly data we use smoothes out the effects of weekly promotions.

2.3.2.4. Sales seasonality

This parameter affects sales at two levels. Some product categories are highly seasonal and retailers anticipate sales peaks and allocate additional space in order to respond to anticipated demand. In such cases, seasonality modifies the causal relationship between sales and space. On the other hand, some stores have a seasonal demand because they are located in tourist areas; the direction of causality is not likely to be affected by this.

3. Empirical estimation of the sales–space relationship

3.1. A share of space vs. share of sales model

3.1.1. Database and methodology

A sales–space relationship is estimated from a pooled database of more than 200 stores belonging to the same French town-center variety store chain. Variety stores are self-service stores, generally located in the center of towns, which supply food and non-food products (non-food departments represent between one and two-thirds of total turnover). They average between 400 and 2500 m^2 of surface area. The database includes a year of monthly sales, space and margin data by category. The variance in space and sales data results from both time-variations within one store and variations among stores.

The hierarchical structure of the assortment, which clusters products proximate in terms of demand and cost, is

organized in six levels (Fig. 3). The highest level is the general level of the store. The second level is for general use products (most frequently bought items) separated from other products. Below this, non-general use products (departments 0–60) are classified into three groups: new products – textile – fashion (group 1), home – do-it-yourself – household linen (group 2) and fancy goods (group 3), and general use products (departments 60–90) are classified into household products (group 4) and food products (group 5). Product categories are at the fourth level: our analysis focuses on this level. Finally, product categories are divided into sub-product categories and SKUs.

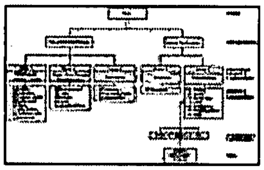

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Fig. 3. Hierarchical structure of the retail chain assortment.

A typology constructed from 50 store characteristics (size, position, presence of a car park, etc.) is used by the retail chain. Norms are computed separately for each of the following store groups: the essentials, the standards or the plus stores. The underlying hypothesis is that they have different profiles in terms of specific characteristics. Those groups are linked to the size of the store: with the *standards* (group g=2) as a reference index (100), the *essentials* (g=3) have an average surface index of 64 and an average turnover per square meter of 77. For the *plus* (g=1) the indexes are 135 and 164. Some stores which do not have both a food and a non-food department were disregarded, giving a final sample of 126 stores (35 *plus*, 65 *standards* and 26 *essentials*).

In practice, in the retail chain examined, the store manager determines space allocation between product categories. But this decision is subject to review, as the retail chain headquarters regularly checks allocation efficiency by using the computed norms. Compared to other retail chains, store managers seem to have a great deal of discretion in allocating space to product categories. The variability in space allocation is, therefore, sufficiently great that the available data can be considered to provide a solid basis for elasticity evaluation (for more detailed data on the variability of space allocation, see A and B).

3.1.2. The model

The model is based on a demand function linking the share of sales to the share of space allocated to a product category. Demand is measured by the monthly turnover and shelf space by the number of linear meters (length of the shelves on which the product category is presented). Space is not weighted to incorporate location value within the store. This simplifying hypothesis was also made by Corstjens and Doyle (1981) and Corstjens and Doyle (1983), Bultez and Naert (1988), Bultez et al. (1989) and Bultez et al. (1995).

A proportional model is chosen to eliminate the direct effect of the store size. This model is closer to a retailer's view because the 100% of store space must be allocated among product categories.

The demand function links the share of sales to the share of space allocated for a product category (c) in store (s) belonging to store type (g) defined above. The multiplicative power model allows interactions between parameters with constant elasticities. Coefficients are estimated (OLS) per product category:

$$\mathrm{SSALES}_c = \exp\ (\alpha_{o_c}) \times \mathrm{SSPACE}_c^{\beta_{cg}} \times \exp\left(\sum_{i=1}^{125} \delta_{i_c} s_i + \sum_{j=1}^{11} \gamma_{j_c} m_j\right) \quad (1)$$

with SSALES_c: turnover share of the product category (c); SSPACE_c: share of allocated space to the product category (c) measured in linear meters; β_{cg}: space elasticity of the product category (c) for the store group (g); $\delta_{i_c} s_i$: dummy for the store (s_i) and the product category (c); $\gamma_{j_c} m_j$: dummy for the month (m_j) and the product category (c).

Because of the homogeneity of the database, two store parameters are given: belonging to a store chain, and retailing format. We use a store group interaction term to the share of space variable in order to ascertain differences of responsiveness to space allocated across the store types defined by the chain (which have different patterns of specific store characteristics). Location and competitive environment are not taken into account as such in this model because the database does not contain this information. However, dummy parameters for the stores are added to allow for their differences.

For individual characteristics, we will test the hypothesis that space elasticity is related to impulse buying by comparing highly space-elastic product categories with high impulse buying categories. Assortment characteristics and promotions are not taken into account. This simplification is justified in Section 2.

To reduce the effect of the causal relationship between sales and space resulting from retailers anticipating seasonal demand by allocating space, we have eliminated highly seasonal categories (swim wear, stationery, toys, etc.) from our analysis. Moreover, we introduced dummy parameters for each month to allow for any remaining seasonal space allocations made in anticipation of demand. This helped to reduce the causal effect of sales on space. Product categories that were absent from most stores (bread, pastry, fish, etc.) were also eliminated to obtain estimates calculated with a sufficient number of observations. Consequently, 24 of 43 categories are included.

3.2. Results

Statistical estimates of space elasticities are satisfactory as the *F*-tests are all significant to about 0.01%. High values for R^2 are obtained ranging from 0.8034 to 0.9535 for product categories (the average R^2 is 0.8884).

The distributions of space elasticity for the different types of stores are given in Table 1. The average value for the space elasticity (weighted by the number of stores of each type) is 0.2138. This value is very close to the results in the literature at SKU level.

Table 1. Distribution of space elasticities across store types

	Plus stores	Standard stores	Essential stores
Mean	0,2496	0,2225	0,1437
Standard deviation	0.2245	0.2076	0.287
Minimum	−0,12	−0,13	−0,44
Maximum	0,68	0,57	0,80

Space elasticities[1] for each product category vary considerably, from −0.44 to 0.80 (Fig. 4). If we take the *standard* store type as a reference (because it is the most common and centered in terms of size), the highest values of the space elasticities are found for costume jewelry (0.57), fruit and vegetables (0.57), underwear

(0.55), shoes (0.50) and haberdashery (0.49). Looking at the lowest values, there are a significant number of product categories with elasticities which do not differ notably from zero at 0.1% level (2 in *plus* stores, 5 in *standard* stores and 10 in *essential* stores): a share of space increase does not result in any change in share of sales. These unresponsive categories cover mainly textile, do-it-yourself and kitchen. More surprisingly, we found negative space elasticities (−0.12 and −0.13 for the fashion category, respectively, in *plus* and *standard* stores; in *essential* stores, four elasticities are significantly negative). It is counter-intuitive unless these categories remain, despite their poor performance, for reasons linked to the point of sale image, in order to attract people in the store. We can, therefore, assume that in some stores, for product categories with negative elasticities, criteria other than sales optimization (linked to the margin level or to the image-building nature of some product categories) are taken into account to allocate space; this results in allocating more space than a simple space optimization would entail.

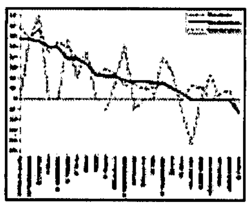

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Fig. 4. Space elasticities.

Store profiles for *plus* and *standard* stores are very similar, as can be seen from Fig. 4. The 95% confidence level for the average comparison test for these two store types does not eliminate the hypothesis that the average space elasticities are equal. For small stores, statistical data is not sufficiently stable to be conclusive. If we exclude essential stores, therefore, the very close correlation between the pattern of elasticities for *plus* and *standard* stores does not confirm the hypothesis that the store type influences the general level of space elasticity.

Table 2 shows the space elasticities found for the product categories highlighted by Bellenger et al. (1978) as being subject to frequent impulse buying, and included in our product nomenclature. Space elasticities for costume jewelry and shoes (in *plus* and *standard* stores) are actually very high. For recording and books, space elasticities are slightly below average in *plus* and *standard* stores but high in *essential* stores. For ladies' wear, however, elasticities are very low. We were unable to compare the food departments considered by Bellenger et al. (bakery goods, meals and snacks, that were assumed to generate considerable impulse buying) with ours, as they were very different in nature. This comparison is subject to reservations but it does provide some support for the hypothesis that impulse buying products are more space-elastic than staples.

Table 2. Space elasticities of high impulse categories defined by **Bellenger et al. (1978)**

	Plus stores	Standard stores	Essential stores
Costume jewelry	0.68	0.57	NS
Shoes	0.46	0.50	NS
Recording/books	0.13	0.18	0.53
Ladies' wear	0.13	NS	−0.44

The main impulsive categories listed by Bellenger et al. have rather high space elasticities; we must also

examine other highly elastic product categories. In *plus* and *standard* stores, the other product categories that react strongly to space allocated are fruit and vegetables (and to a lesser extent other food categories), underwear, haberdashery and perfumery. The marketing manager has confirmed that they can also be considered to encourage impulse buying.

The pattern of elasticities is, therefore, consistent with the level of impulse buying across product categories. Our hypothesis that there is a positive relationship between space elasticity and impulse buying is confirmed.

4. Managerial implications and future research

Our results support the hypothesis that direct space elasticities are significantly non-zero for most product categories (average space elasticity is 0.2051, in line with the literature at SKU level). They also support the hypothesis that elasticities vary greatly from one category to another. We have suggested that various store and category characteristics might explain the sensitivity to space allocated. Of these characteristics, we tested the influence of the impulse buying rate for the product category. The results show that impulse buying categories have higher space elasticities, which is consistent with the interpretation that space has a causal effect on sales and not the converse. The influence of the store type (measured by the store group interaction term to the share of space variable), as defined in the retail chain's own typology, is not significant.

Several reservations limit the possibilities for generalizing the results. First, although different store types were studied, the database was limited to a single retailing format and to one retail chain for 1 year. Further research is needed to verify the reliability of the space elasticities estimates (levels and patterns) for other formats such as hard discounters, hypermarkets, specialized stores or *category killers*, over a longer period of time. Second, the results are drawn from an econometric model applied to cross-sectional and time-series data. The major advantage of this methodology for the store chain is that space elasticities can be computed from available data and regularly re-estimated. Nevertheless, this model does not permit a causal link between space allocated and sales to be established: the causality issue remains central.

One direct consequence of the wide range of estimates for the space elasticities across the product categories is that space allocation rules based on constant space elasticities equal to 1 (as assumed by the current norm-based diagnostics system and commercial software) or fixed at a constant value such as 0.5 (as conventionally assumed for mail order selling) do not provide efficient space allocation. Present retailers' practice leads to less than optimal allocation because the differences of product category sales responsiveness to shelf space allocated are not taken into account.

The current retail chain norm-based diagnostics system should be modified to use space elasticities to weight product categories in the computed norms. In a related research field, Albers (1997) showed that marketing budgets allocated across products or market segments proportionally to profit contribution multiplied by its elasticity leads to near optimal results. In the same way, space allocation optimization programs taking space elasticities into account are required. Moreover, the introduction of space elasticities in the diagnostic system should improve the reliability of allocation recommendations because the consequences of space allocation changes on sales can be forecast through the leveraging effect of sales sensitivity.

We have focused on the estimation of direct effects of space allocated on sales and we disregarded cross-elasticities, contrary to recent literature. This choice was justified by the fact that cross-elasticities are weaker than direct effects. Our goal was limited to emphasizing the wide range of space responsiveness and providing some explanation of this diversity. In a second stage, cross-effects have to be measured to thoroughly

understand category sales responsiveness to space. We believe that it is important to detect traffic-building product categories which create additional demand for the entire store by measuring the elasticity of total store turnover to space allocated to each product category. The underlying assumption is that cross-elasticities between product categories result less from direct demand substitution or complementarity than from the power of attraction of some traffic-building product categories.

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Appendix A: Distributions of the share of sales in each product category

	Plus stores		Standard stores		Essential stores	
	Mean	Standard deviation	Mean	Standard deviation	Mean	Standar
02	11.22	0.63	1.11	0.6	1.62	
03	4.05	1.62	4.03	1.38	5.63	
05	0.41	0.29	0.32	0.2	0.51	
12	3.23	1.44	3.6	1.57	5.04	
14	0.48	0.31	0.5	0.3	0.56	
15	1.04	0.36	1.16	0.38	1.64	
16	1.69	0.71	1.88	0.83	2.77	
17	2.87	1.18	2.77	1.11	4.09	
22	0.91	0.25	1.08	0.37	0.96	
28	0.68	0.27	0.61	0.27	0.59	
30	2.35	0.38	2.31	0.48	2.17	
32	0.50	0.22	0.54	0.27	0.6	
36	0.82	0.45	0.89	0.5	1.13	
42	8.62	2.88	9.08	2.52	10.64	
45	2.81	1.03	2.94	1.36	3.63	
51	0.40	0.24	0.55	0.23	0.83	
63	2.63	0.64	2.78	0.8	2.38	
61	17.46	2.1	18.31	2.56	17.98	
67	10.60	1.98	10.86	2.85	8.32	
73	3.14	0.64	3.26	0.84	2.48	
79	12.67	1.82	12.39	1.67	10.65	
83	6.54	1.21	6.99	1.29	6.64	
84	7.69	1.9	7.54	1.96	6.59	
86	8.91	1.75	9.83	2.23	9.17	
All	4.32	4.65	4.38	4.79	4.44	

Data is missing for some product categories in some stores; consequently, total sales add up to more than 100%.

Appendix B: Distributions of the share of space allocated to each product category (in linear meters)

	Plus stores		Standard stores		Essential stores	
	Mean	Standard deviation	Mean	Standard deviation	Mean	Standarc
02	2.79	1.03	2.62	0.94	2.71	
03	8.03	2.19	8.08	1.48	8.69	
05	0.68	0.25	0.54	0.28	0.57	
12	6.52	1.96	6.24	1.40	6.40	
14	1.51	0.84	1.54	0.74	1.54	
15	4.06	1.12	4.25	0.92	4.74	
16	6.54	1.63	6.65	1.34	7.53	
17	7.06	1.77	6.72	1.64	7.18	
22	1.66	0.85	1.76	0.71	1.52	
28	1.69	0.47	1.73	0.65	1.56	
30	3.78	1.09	4.72	1.55	3.83	
32	1.66	0.94	1.81	1.13	2.14	
42	8.49	1.87	9.12	1.61	10.27	
45	4.81	1.23	4.40	1.52	4.22	
51	0.78	0.42	1.14	0.47	1.39	
63	2.30	0.77	2.79	0.79	3.10	
61	14.39	4.25	14.81	3.21	15.72	
67	5.65	1.93	5.23	1.49	4.51	
73	3.47	1.52	3.37	1.20	3.08	
79	4.07	1.28	3.81	1.00	2.84	
83	3.04	1.00	2.82	0.93	2.14	
84	4.46	1.70	4.27	1.33	3.73	
86	2.36	0.90	2.16	0.87	1.59	
All	4.32	3.44	4.38	3.41	4.44	

Data is missing for some product categories in some stores; consequently, total space adds up to more than 100%.

Appendix C: Space elasticities for each product category and store type

	Adj. R^2	Plus stores	α^a	Standard stores	α^a
02 Faction	0.9045	−0.12	0.0001	−0.13	
03 Underwear	0.8238	0.67	0.0001	0.55	
05 Costume jewelry	0.8044	0.68	0.0001	0.57	
12 Ladies' wear	0.9012	0.13	0.0001	0.01	
14 Shoes	0.8176	0.46	0.0001	0.50	
15 Baby clothes	0.8904	0.12	0.0001	0.03	
16 Children's wear	0.8985	0.04	0.1587	−0.10	
17 Men's wear	0.9000	0.06	0.0010	0.05	
22 Do-it-yourself	0.9090	0.08	0.0001	0.02	
28 Table	0.8034	0.11	0.0008	0.16	
30 Kitchen	0.8792	0.03	0.0082	0.06	
32 Home	0.8205	0.10	0.0001	0.11	
36 Household linen	0.8412	0.15	0.0001	0.16	
42 Perfumery	0.9039	0.50	0.0001	0.39	
45 Recording/Books	0.9422	0.13	0.0006	0.18	
51 Haberdashery	0.9085	0.54	0.0001	0.49	
63 Household	0.9204	0.15	0.0001	0.17	
61 Grocery	0.9123	0.29	0.0001	0.22	
67 Drinks	0.9381	0.39	0.0001	0.39	
73 Frozen food	0.9177	0.10	0.0001	0.17	
79 Dairy	0.9535	0.23	0.0001	0.23	
83 Delicatessen	0.8949	0.25	0.0001	0.22	
84 Fruit/Vegetables	0.9042	0.58	0.0001	0.57	
86 Meat	0.9329	0.39	0.0001	0.33	

[a]Probability that the coefficient is not significantly different from zero.

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[1]Complete results on space elasticities can be found in C.

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A study on shelf space allocation and management

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Abstract

Shelf space is one of the most important resources to attract more consumers in logistic decisions. This paper proposes a comprehensive space allocation model with an alternate form that is applicable to the retail practice. Space management strategies for the retailers are also presented and interpreted through the analysis of data collected from a questionnaire survey. Finally, the issue of establishing a computer system for space management is also discussed.

Author Keywords: Logistics; Information processing; Space allocation

Article Outline

1. Introduction

Shelf space is one of the most important resources to attract more consumers in logistic decisions. Managing shelf space well can not only decrease inventory level but also have stronger vendor relationship and higher customer satisfaction as well [1]. The decision of shelf space allocation and management is therefore a critical issue in retail operations management.

A shelf space allocation problem is a decision problem to reach the possibly best objective under some operational constraints in a retail store. It can be considered as an extension of knapsack problem. Owing to the complexity to obtain its optimal solution, intuitive approaches have largely been used in commercial space management systems. These commercial systems use relatively simple heuristic rules to develop operational guidelines designed to permit retailers easily to implement decisions of shelf space allocation in practice [2]. The concern of simplicity and practicability of these commercial approaches will certainly have a negative effect on their solution performance. However, high-speed computers of low cost are nowadays available to overcome, in some degree, the problem of complex computations. The optimization approaches are therefore applicable to be used in space management systems for improving the performance of shelf space allocation.

After reviewing the related literature, this paper proposes a comprehensive optimization model for allocating shelf space. This model is then modified to an integer programming model for increasing its applicability in the retail practice. As the management strategy may affect the operations of space allocation and their performances, this study then attempts to explore the relationships among them based on the result of a questionnaire survey. At last, this paper will also describe how to establish a computerized decision support system to fulfill the task of effective shelf space allocation. According to space management strategies that retailers adopt, different approaches for system development will be suggested.

2. Related studies

There are five aspects of shelf space allocation which management can consider to improve the financial performance of the store. They are fixture location, product category location, item location within categories, off-shelf display, and POS promotional support [3]. Two commonly used techniques in commercial systems for allocating shelf space are the sales productivity method and the buildup method [4]. Space elasticity is another approach used in many experimental studies. It was defined as the ratio of relative change in unit sales to relative change in shelf space [5]. Doyle and Gidengil [6] had reviewed in-store experimental approaches, summarized the results from many studies about space elasticity and pointed out the difficulties that might be encountered as these approaches be applied in retail practice. Drèze et al. [7] made a series of field experiments and found that location had a large impact on sales, whereas changes in the number of facings allocated to a

brand had much less impact as long as a threshold was maintained.

Commercial approaches and experimental approaches fail to evaluate the aggregate store performance of their allocation solution. Therefore, optimization models with an application orientation are worthy of consideration. A model that can determine the optimal brand collection and display area allocation simultaneously was proposed by Anderson and Amato [8]. The optimization model included the analysis of main demand effect but cost effect was ignored. The mathematical programming model of Hansen and Heinsbroek [9] incorporated main demand effect with cost effect and made the model more complete. However, neither of these two studies considered the cross effect among products within the store. Corstjens and Doyle [10] developed a more comprehensive model, considering the main effects and the cross effect of demand and the cost effects, which was a generalized geometric programming problem that can be solved by a practicable algorithm proposed by Gochet and Smeers [11]. Its objective function was the total profit of the retailer that was composed of individual product demand and cost functions in forms of general polynomials. The optimal objective was to be solved under four sets of constraints: store capacity constraint, availability constraint, control constraints of lower and upper bounds, and nonnegativity constraints.

As experimental studies have argued that space might have less effect on demand and sales than other marketing variables, Zufryden [2] proposed a model with a general objective function accounting for space elasticity, cost of sales, and potential marketing variables that relate to demand. A dynamic programming approach for this model was also presented. Corstjens and Doyle [12] also provided a dynamic model that strategically incorporates the goodwill or carryover effects retailers accrue from operating in a given merchandise area over a period, and can encourage the retailers to allocate more space to new products and divest earlier from declining ones.

However, there exist some limitations to practically apply the models of Corstjens and Doyle [10, 12] and Zufryden [2]. The geometric programming model of Corstjens and Doyle ignores the integer nature of numbers of displayed products. This makes its solution probably suboptimal. In addition to the same limitation, their dynamic model is merely formulated as a set of equilibrium equations and is too theoretical and complex to be applied in retail practice. The dynamic programming model of Zufryden requires the display area of each product to be multiples of the area unit values, hence limits the flexibility of retailers' decisions on shelf space allocation. Besides the drawbacks discussed above, the location effects which Drèze et al. [7] emphasized are neglected by current optimization models.

3. Space allocation models

3.1. A comprehensive model

Suppose there are m shelves, with length T_k for shelf k, in a store. The length of a facing of product i displayed on any of these shelves is a_i, i=1,..., n. The location effect along with the assumptions in models of Corstjens and Doyle [10, 12] and Zufryden [2] are considered. Let x_{ik} be the amount of facings which product i displayed on shelf k, the demand function can be defined as

$$Q_{ik}(x_{ik}) = \alpha_i x_{ik}^{\beta_{ik}} \prod_{\substack{j=1 \\ j \neq i}}^{n} x_j^{\gamma_{ij}} \prod_{t=1}^{L} y_{ti}^{\delta_{ti}} \tag{1}$$

where α_i, β_{ik}, γ_{ij}, δ_{ti} are parameters, β_{ik} is the space elasticity of product i on shelf k, γ_{ij} is the cross elasticity between products i and j, δ_{ti} is the elasticity of product i relative to tth (t=1,..., L) marketing variable. γ_{ij} need not be equal to γ_{ji}, and γ_{ij} can be 0, positive, or negative depending on product i and j being independent, complementary, or substitute. $x_j=\sum^m_{k=1}x_{jk}$ is the total amount of facings of product j, y_{ti} is the amount of tth marketing variable, other than shelf space, allocated to product i. The marketing variables possibly include price, advertisement, promotion, store characteristics, and other marketing mix variables. Q_{ik} is actually a function of x_{ik}, y_{1i},..., y_{Li}, which is represented in a simple form of $Q_{ik}(x_{ik})$:

$$Q_{ik}(x_{ik})=Q_{ik}(x_{ik}, y_{1i},\dots, y_{Li}) \tag{2}$$

The objective is to maximize the total profit P that is a function of x_{ik} for fixed values of y_{1i},..., y_{Li}. The total profit can be obtained by deducting the display-related overhead from the gross profit of the entire store. The gross margin of an item equals to its unit price minus its buying cost and the display-related overhead includes ordering cost, and holding cost of this item. The components of ordering cost are order processing expense, transportation expense, loading and unloading cost. Holding cost consists of the expenses of space, interest, insurance, damage, and personnel that occurred in the warehousing operation. Assume that the gross margin of product i is linear to its unit margin g_i, the sales volume elasticity associated with the variable cost of product i is η_i. Then the gross cost $c_i(x_i)$ of product i can be calculated by

$$c_i(x_i) = \theta_i\left(\sum_{k=1}^{m} Q_{ik}(x_{ik})\right)^{\eta_i}. \tag{3}$$

The equation of total profit is

$$P = \sum_{i=1}^{n} g_i\left(\sum_{k=1}^{m} Q_{ik}(x_{ik})\right) - \sum_{i=1}^{n}\theta_i\left(\sum_{k=1}^{m} Q_{ik}(x_{ik})\right)^{\eta_i}, \tag{4}$$

where θ_i is a constant cost coefficient of product i.

There are four sets of constraints. First is the capacity constraint T_k of shelf k. Second, the lower and upper bounds of facings of product i are set as U_i and L_i separately. Third is the availability constraint that sales of product i can not exceed its supply limit A_i. Last is the constraint that the decision variable x_{ik} is a nonnegative integer.

The model can be formulated as follows:

$$\begin{aligned} &P = \sum_{i=1}^{n} g_i\left(\sum_{k=1}^{m} Q_{ik}(x_{ik})\right) - \sum_{i=1}^{n}\theta_i\left(\sum_{k=1}^{m} Q_{ik}(x_{ik})\right)^{\eta_i},\\ &\text{subject to}\\ &\sum_{i=1}^{n} a_i x_{ik} \leqslant T_k, \quad k = 1,\dots, m, \end{aligned} \tag{5}$$

$$L_i \leqslant \sum_{k=1}^{m} x_{ik} \leqslant U_i, \quad i = 1,\dots, n, \tag{6}$$

$$\sum_{k=1}^{m} Q_{ik}(x_{ik}) \leqslant A_i, \quad i = 1,\dots, n, \tag{7}$$

$$x_{ik} \in N \cup \{0\},\ i=1,\dots,n,\ k=1,\dots,m. \tag{8}$$

The model is a nonlinear programming that could be very complicated to solve for retailers like supermarkets that sell a great variety of different items. It is therefore worthwhile to consider its alternate form to make it easier to solve and more applicable in practice.

3.2. An alternate form

Though unlimited supplies of product may not be always available, the retailers can prevent themselves from out-of-stock occurrences by building effective logistic systems. Then the availability constraint, Eq. 7, can be ignored. Furthermore, in case that the sales volume elasticity η_i associated with the variable cost of product i is unable to be estimated, it can be approximately assumed that the profit of product i is linear within a small range of the amount of facings that product i is displayed. The condition of this approximation can be achieved by controlling the size of U_i and L_i.

Under the circumstance that these assumptions hold for all n products of the retail store, let p_{ik} be the per facing profit of product i on shelf k. The shelf space allocation model can be reformulated as follows:

$$\max P = \sum_{i=1}^{n}\sum_{k=1}^{m} p_{ik}x_{ik} \tag{9}$$

subject to Eq. 5, Eq. 6 and Eq. 8.

The alternate form is an integer programming and is more applicable since there are currently many computer packages that can be used for its solution. Though it is still a NP problem, a multi-stage solving procedure is also presented to reduce the complexity to solve it. The multi-stage solving procedure divides the problem into some subproblems with much smaller size, thus make the solution task can be completed easily.

Usually, there are a lot of product items being considered whether or not to be displayed in a retail store. For supermarkets in Taiwan, the item amounts of merchandises are ranged from 8000 to 12 000. To put so many items together into a space allocation model is not only infeasible in computation but also meaningless, in practice. The retailers often categorize their product items into several levels to manage efficiently the store operation. The first level is usually called department, such as biscuit department. The second level is usually called category, such as chocolate biscuit category. The third is usually the lowest category level and called item, such as a specific brand of chocolate biscuit. This categorization can be executed in different way to fit the store policy and more level of subcategories can be extended if necessary. Following the product categories, merchandise manager can determine the product mix of the store level by level from the top to the bottom. The multi-stage solving procedure solves the space allocation model in the same way.

Using the multi-stage solving procedure, the operation manager first allocates the shelf space to the product categories on the first category level. The optimal allocation amount of each category on the first level can be found by solving the space allocation model. At the second stage, there is a subproblem to be solved for every category on the first level. The allocated amount of space of each category obtained from the first stage is then be treated as available space (capacity constraint) of the corresponding subproblem, which can also be formulated as a space allocation model. The allocation amount of each category on the second level thus can be solved. Similar procedures can be continued to lower stages until the space had been allocated to overall product items of entire store. Obviously, the final solution after completing the multi-stage solving procedure is a

suboptimal one although the optimality exists in every single subproblem. Nevertheless, it can be regarded as the optimal decision. This argument is supported by two points. One is because there is an actual need of managing merchandise through categorization in retail practice. Another point is that the extreme computational complexity to solve the space allocation model makes the search for true optimality unrealistic.

In addition to matching the need of managing merchandise through categorization in retail practice, the most important advantage of the multi-stage solving procedure is that it makes the enormous computational work of solving space allocation model feasible, both operationally and economically. Take a retail store selling 8000 items for instance. The merchandise of this store has been categorized evenly into three levels: departments, categories, and items. Suppose that there are 20 departments, with 20 categories in each department and 20 items in each category. Using the multi-stage solving procedure, there will be one subproblem to be solved on the first stage, 20 subproblems to be solved on the second stage, and 400 subproblems to be solved on the third stage. The totals are 421 subproblems,whose problem size are 20 items only. If a space allocation model for 20 items needs C times of computation, it will take about $421C$ computation times by this approach. On the other hand, if we put whole 8000 items into the space allocation model at one time, since the size of this problem is 400 times larger than that of each subproblem, the computation times will become about C^{400} due to the nature of exponential computation times of the model.

This model was tested by a problem of six items allocated to six shelves. There were 36 decision variables and 18 constraint equations other than nonnegativity and integrality constraints in this integer programming problem. It was solved by the QSB+ in an IBM compatible P.C. with a CPU of Pentium 75. It spent near 11.5 hours to solve this problem. However, the six items could be divided into two categories with each category has three items and the shelf space was first allocated to the categories and then to the items, that is, the shelf space allocation problem was solved by two stages. The first stage was an aggregate problem on the level of product category, and the second stage consisted two subproblems corresponding to the category. It only took total 0.4 seconds (7500 times faster) to solve these three problems generated from the approach and the decrease of the optimal value was 1% only. As it is a regular practice to allocate shelf space according to the categories of merchandise, the model is therefore applicable to retail stores.

3.3. Parameter estimation

When the proposed model is applied to the practice of shelf space allocation, the first problem an operations manager of a retail store needs to cope with is how to construct an appropriate model that can well represent the actual conditions of the store. Correspondingly, this is also a problem of determining the values of all parameters of the shelf space allocation model. The values of parameters of the demand functions (Eq. 1) and of the cost functions (Eq. 3) may be estimated by statistical analysis. The upper bounds and lower bounds for amount of facings of products are determined by the management according to the policy of the store. Other remaining parameters, such as unit profit margin, length of facing, capacity of shelf, and supply limit, can be obtained from the actual data.

There are two approaches to estimate the parameters of the demand functions and of the cost functions. One is using regression analysis of the actually operational data and the other is using experimental design. When the available operational data is not sufficient for estimating the parameters, experimentation is a very good way to provide substitute information. The main advantages to use experimental design for parameter estimation are its rigidness of methodology and its capability of generating almost all kinds of relevant data for modeling shelf space allocation. However, the cost of field experimentation is so high that hinder it from being used in the situation that involves a large amount of parameters. To overcome the cost limitation, computer simulation is an

alternative method for executing shelf experiment economically.

For a retailing enterprise such as chain stores that have a lot of cross-sectional data from different stores, regression analysis may be a convenient method to estimate model parameters. Since store location is one of the factors that influence the allocation of shelf space in a store, the actual allocation data of shelf space will differ across stores. Based on the difference of the amount of shelf space allocated to a product in different stores, the effect of shelf space allocation on demand of the product can therefore be estimated by the regression analysis. The effects of the other marketing variables on product demand may be included in the same regression model, and the relationship equation of cost to demand can also be estimated by the same way. However, this cross-sectional analysis may not always be applicable, especially for those small scale retailers. In those cases, longitudinal analysis seems to be a reasonable alternative, and regression analysis may be conducted on the accumulated operational data over a period of time.

Having the values of parameters determined, the model will be solved and the resulted solution can be used as the operational guideline for the allocation of shelf space. The allocation operations may then have great impact on the space-related performances. But as the model contains some policy constraints, the allocation operations may also be influenced by the strategy of space management. The potential relationship among them is worthy of study and will be discussed in the next section.

4. Management strategies

4.1. Methodology

Space allocation decision is very important in retail operations and management. Considering the organizational and environmental factors a retail firm encounters, the management will determine the strategy for space management of the store. Once the decision of space management strategy has been made, an appropriate allocation model might be adopted in the shelf space operation and possibly affects the resulted performance. This relationship among space management strategy, space allocation operation, and performance, which can be represented as Fig. 1, will be explored by a questionnaire survey.

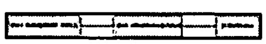 **Display Full Size version of this image** (3K)

Fig. 1. Conceptual framework of the questionnaire survey.

Three testable arguments can be derived from Fig. 1. The null hypotheses to be statistically rejected can be stated as follows:

H1.	For retailers that adopt different strategies for space management, their operation
H2.	For retailers that adopt different strategies for space management, their performa
H3.	For retailers that adopt different operations of space allocation, their performanc

Fig. 1 also shows that there were three constructs involved in this survey. First, the construct of space management strategy was composed of four variables: the goal of space management, the plan of space management, the organizational level where the decisions of space management were made, and the degree of

significance of space management to the corporate management. Each variable was measured by a Likert scale of five points. The second construct was the operation of space allocation that were measured by five levels of operational activities: planogramming, inventory control, sales analysis, merchandise planning, and category management. According to the order of these stages, the operation of space allocation of the store was scored from one to five, respectively. The third construct was performance. There were a lot of indexes that can be used to measure the performances of the retail stores from the related literature. Having considered the result of interviewing with the store managers of 13 supermarkets, this study selected sales turnover, sales per square feet, and gross profit per square feet as the performance variables to be empirically measured in questionnaire survey.

Questionnaires about shelf space management were mailed to supermarkets in Taiwan. There were 844 supermarkets in the roster of Taiwan Supermarket Association and 73 of them were not available. Since the population was not too large, the 771 supermarkets are all included in the sample. The questionnaire asked the respondents to provide the performance data that they considered to be private, therefore some returned data were incomplete or inconsistent. After data preparation, only 67 responses (8.7%) could be used for further analysis. However, the test result of goodness-of-fit showed that this sample is a good representation of the population at a significance level of 0.05.

The reliability of questionnaires was measured by Cronbach's α coefficient, whose value was 0.81. So the results of this survey were quite reliable. The responded data were analyzed by cluster analysis, analysis of variance (ANOVA), and multiple comparison test. Cluster analysis was used for the classification of strategies for space management. ANOVA was mainly used to test the difference of operations and performances between the stores that adopted different strategies. Sheffe test was used for multiple comparison between the stores when the result of ANOVA was significant.

4.2. Results and analyses

This study used two-phase clustering method to do cluster analysis. Table 1 shows that the strategies of shelf space management can be classified in three types: the dominance strategy, the adaptation strategy, and the passiveness strategy. From the table, we can see the average value of the strategy and find that the dominance strategy is the best, the adaptation strategy is second, and the passiveness strategy is the worst. The strategy clusters also had different average value of the level of operation for space allocation. Still, the dominance strategy had the highest and the passiveness strategy had the lowest. For the three performance indexes, the same order of rank could be found from the results of ANOVA too.

Table 1. Strategy clusters

Name	Observations	Value
Dominance strategy	33	17.06
Adaptation strategy	16	14.63
Passiveness strategy	18	12.67

To find the relationship among the strategy of space management, the operation of space allocation, and performance, three hypothese were tested by ANOVA. The F value for hypothesis H1 was 8.12 (p=0.007) and the test result showed that hypothesis H1 was rejected. In other words, we can say that, for retailers that adopt different strategies for space management, their operations of space allocation are very different. Moreover, the result of Sheffe test showed that the cluster of dominance strategy had the highest level of operation for space allocation, the cluster of adaptation strategy and passiveness strategy followed accordingly.

Sales turnover, sales per square feet, and gross profit per square feet were used as the performance variables to test hypothese H2 and H3. Table 2 summarises the results of ANOVA. When sales per square feet or gross profit per square feet were used, both hypothese H2 and H3 were rejected. However, both hypothese can not be rejected when sales turnover was used. That is, considering the performances of sales per square feet and gross profit per square feet, we can argue that, for retailers that adopt different strategies for space management, their performances are different. For retailers that adopt different operations of space allocation, their performances differ. This argument fails when sales turnover is the measure of performance.

Table 2. Results of ANOVA

Hypothesis	Sales turnover	Sales per square feet	Gross profit per square feet
H2	2.27(0.113)	3.20(0.048)	9.87(0.000)
H3	1.43(0.235)	6.18(0.000)	6.65(0.002)

The numbers before and in the parentheses are F values and p values, respectively.

Sheffe tests were executed to compare the performances among the strategies for space management and among the operations of space allocation after the null hypothese had been rejected by ANOVA. Either sales per square feet or gross profit per square feet was used as the measure, the dominance strategy had the best performance. The adaptation strategy was the next and the passiveness strategy was the worst. The performance ranks among the operations of space allocation were consistent for these two performance indexes. When the operation of space allocation reached the level of category management, performance was the best. The following was merchandise planning, sales analysis, inventory control, and planogramming in the order of performance rank.

We can concluded from the above analysis that, with significance level α=5%, the management strategies may affect the operations of space allocation and their performance, and the operations of space allocation may influence their performances as well. Some strategical implications to the retailers can also be obtained from this survey. First, retailers should take a more positive attitude about their strategies of space management. Second, retailers should raise their levels of operation for space allocation. Third, retailers need to reinforce their computerization for space allocation and management. Last, retailers should put more emphasis on the space-related indexes of performance for space management.

5. Computerized space management

5.1. Computerized systems

The problem of space allocation is so complex that most retailers do not have enough knowledge to make appropriate decision. It involves tremendous data while solving this problem and therefore demands a computer system for better space management. There are currently numerous PC-based shelf space management systems available to retailers including Apollo (IRI) and Spaceman (Nielsen). Most of the systems are mainly used for planogram accounting purposes but not for the higher level of operational activity. One reason is that the data collection is difficult to the retailer, another reason is due to the limited functions of decision support provided by these systems. The former problem can be resolved by introducing a POS information system, and the latter is a problem of how to build an effective model base that should be put into consideration during system development.

The computer system for space management can be developed in three phases. First is to provide the basic function of planogramming. The goal of this phase is to reach the optimal sales, profit, or other financial

measures. The essential task at this phase is to solve the problem of space allocation optimally. Second is to enter the phase of inventory analysis. At this phase, the interaction between shelf space and inventory should be considered. The function is to analyze the related operations of purchasing, shipping, loading and unloading, stocking, and picking for improving the performance of inventory control. In addition to the basic goal of the first phase, the goal of inventory such as sales turnover must be added to this stage. Third is the phase of category management. There are some new functions at this phase. The analysis of sales trend, the performance appraisal of categories, and the development of promotion strategy can be included as part of system functions. This phase should put emphasis not only on the performance of a single store but also on the integrated performances of the corporation as a whole. The relationship with the manufacturer should also be evaluated.

There are many factors to be considered at the third phase. The space management systems of this phase are extremely complicated and most of them are developed and used by the manufacturers for providing relevant information to the retailers. Nevertheless, for large retailers who want to increase the sales profit, raise the customer satisfaction, and to improve the supplier relationship, to expand their computer systems gradually to reach the phase often can bring about this kind of strategical effect. Since it needs to spend a lot of time and money to develop a complete system for space management, the decision must be very carefully made after thorough evaluation in order to keep away from failure.

5.2. Strategy and system development

The strategy of space management will have an impact on the development of its computerized system. For retailers with dominance strategies, the management regards shelf space as a very important strategic decision and more complete functions of space management system are required. This kind of systems can include category management in their functions at the stage of system analysis and be developed step by step. For retailers with adaptation strategies, the management does not emphasize the space decisions, and the requirements of their space management systems are determined according to the result of system analyses. For retailers with passiveness strategies, the management can not appreciate the importance of space decisions and only the basic functions like planogramming will be included in their space management systems.

In the process of developing space management systems, retailers need to spent much time to collect a great deal relevant data. Although data like the sizes of shelf, product, and space, rely heavily on input by hand, historical data like price, sales, cost, profit can be automatically collected, calculated, and analyzed by computer. Therefore, the space management system must be able to integrate with the corporate management information system in order that the data the former system need can be supplied by the latter. This integration will not only facilitate the development of the space management but also can increase the system effectiveness by revising the decisions of space allocation and management through the analysis of performance data about implementation feedback from the management information system.

6. Conclusions

After reviewing current available models for shelf space allocation, a comprehensive optimization model was proposed. Since this is a model of nonlinear integer programming, the solving complexity may hinder it from being applied in a retail store with a large number of displayed products. An integer programming model modified from the first one is shown to be applicable in the retail practice. Based on a questionnaire survey, the shelf space management strategies were classified as the dominance strategy, the adaptation strategy, and the passiveness strategy. The management strategy may affect the operations of shelf space allocation and their performances, and allocation operations may influence their performances as well. The retailers therefore should

put emphasis on their space management strategies and raise their levels of operation for space allocation in order to improve their performances. According to strategies of shelf space management retailers adopt, different approaches for developing computerized systems are also suggested.

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SH.A.R.P.: SHELF ALLOCATION FOR RETAILERS' PROFIT

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Shelf space is the retailer's scarcest resource. Space management tools are thus badly needed. For that purpose, commercial computer packages and optimal allocation models have been developed. Building on the work of Corstjens and Doyle, we elaborate a general, theoretical shelf space allocation model, which focuses on the demand interdependencies prevailing across and within product-groups. Rules of thumb are compared to the derived optimal allocation. The SH.A.R.P. model is introduced as a simplified, yet realistic, variant and validated against data collected in a Dutch supermarket chain. Subsequently, the feasibility of its implementation has been experimented in various Belgian chains. Case studies demonstrate the profit potential of SH.A.R.P.

(Shelf Space; Optimal Allocation; Product-Line Interdependencies; Model Implementation)

1. Introduction

In self-service selling situations, display is an important marketing instrument. Consumer goods manufacturers pushing their products into supermarkets worry about both their location on the shelf and the space they fill on it. New merchandising concepts are continuously devised to catch the shoppers' attention such as, e.g., P&G "aisle vision" ads. And recent experiments have dispelled doubts once cast on facing effectiveness (see, e.g., Wilkinson, Mason, and Paksoy 1981 and 1982).

Although a substantial portion of the literature deals with shelf space allocation, Corstjens and Doyle (1982, hereafter C-D) have argued that the problem has not been solved in a conceptually sound, yet operational way. Academic research has focused on demand aspects, while consulting work has emphasized operating cost considerations.

The present article is inspired by the C-D system of interdependent products' sales responses to space. However, our model features several distinctive characteristics. Within product-class sales-share elasticities (direct and cross) are introduced and distinguished from overall product-class elasticities. The sales-share specification selected proves to be robust, easy to estimate, and amenable to simple experimentation. The normative heuristic derived here reveals to be no more difficult to apply than other widely used commercial space allocation packages. In addition, we study to what extent it may outperform the formulae on which those routines are based. The latter are shown to be rather special cases of our own allocation rule.

§2 briefly reviews the relevant literature. The theoretical shelf space allocation model

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TABLE 1
Glossary of Symbols

Shelf-space

S = total shelf space available for the product assortment;

s_i = space allocated to item i, with: $i = 1, 2, \ldots, c, \ldots, n$;

$\sigma_i = s_i/S$, item i's share of the total space available.

Sales

q_i = item i's sales volume, a function of the space allocation;

$Q = \Sigma_i q_i$, the assortment total sales volume;

$m_i = q_i/Q$, item i's share of the assortment total sales.

SR_i = sales revenue for item i.

Cost

C_i = replenishment cost due to carrying item i in the assortment, an increasing function of q_i and a decreasing function of s_i;

$c_i = C_i/\Pi$, relative cost of handling item i;

Profit

g_i = gross profit margin or markup per unit of item i sold, i.e. the difference between its unit selling price and its net purchasing price;

p_i = direct product profitability of item i, i.e. the difference between its mark-up and its marginal replenishment cost;

$\pi_i = p_i q_i$, item i's profitability;

$\Pi = \Sigma_i p_i q_i$, total assortment profitability;

$r_i = \pi_i/\Pi$, item i's relative contribution to the assortment's profitability

Elasticities

$\eta_{ij} = (\partial q_i/\partial s_j)(s_j/q_i)$, cross-elasticity of item i's sales,

$\epsilon_j = (\partial Q/\partial s_j)(s_j/Q)$, elasticity of the assortment total sales,

$\mu_{ij} = (\partial m_i/\partial s_j)(s_j/m_i)$, cross-elasticity of item i's share,

$\gamma_j = -(\partial C_j/\partial s_j | dq_j = 0)(s_j/C_j)$, partial elasticity of item j's replenishment cost, at a constant sales level,

all defined w.r.t. the space allocated to item j.

is discussed in §3. We provide a general economic interpretation for the optimal solution obtained. §4 presents the basics of SH.A.R.P. and relates it with usual rules of thumb. §5 illustrates how it can be implemented. Directions for further developments are indicated in §6.

2. Literature Review

Extensive literature reviews can be found in recent articles and dissertations: e.g., Heinsbroek (1977), Hansen and Heinsbroek (1979), Corstjens and Doyle (1981) and Vanneste (1982). Here we confine ourselves to a "scanning," facilitating the positioning of our contribution. The reader is referred to Table 1 which defines the notation used.[1]

Commercial models actually used by supermarkets determine space allocation following rules of proportionality to sales, revenue or profit. Three examples suffice to illustrate their variants: PROGALI suggests allocation based either on sales volume ($\sigma_i = m_i$) or on sales revenue ($\sigma_i = SR_i/\Sigma_c SR_c$), see Malsagne (1972); OBM relies on gross margins ($\sigma_i = g_i q_i/\Sigma_c g_c q_c$), see Looyen (1970); CIFRINO (1963) recommends dependence on profit ($\sigma_i = r_i$). Although sophisticated computer packages such as SLIM (BCD 1972), COSMOS (1969) and HOPE (Duban 1978) offer more comprehensive space management systems including handling costs and tackling inventory control, none of these explicitly considers sales elasticities to space.

In contrast, the empirically-orientated academic research tradition has been primarily concerned with measuring the *direct* shelf-space elasticity (η_{ii}). Unless the latter reveals significant, space has no merchandising value. As some authors obtained low estimates, they argued that handling cost and avoidance of stockouts should take precedence, e.g. Curhan (1972). Heinsbroek (1977) in reviewing 20 previous studies

[1] All variables are defined with respect to an appropriate time unit. It should also be clear that item sales (q_i and Q) must be defined in physically homogeneous units.

found a rather skewed distribution of elasticity estimates: ranging from 0 to 0.50, with mean equal to 0.15, values inferior to 0.05 in about 40% of the cases, and only 5% greater than 0.45. Such a wide variance may be explained by differences between stores, product attributes, merchandising practices, purchasing habits (extent of unplanned purchasing or impulse buying), see e.g. Brown and Tucker (1961). The relatively high frequency of negligible estimates may indeed be accounted for by poor experimental designs, low variability of space allocation or hasty consecutive changes in shelf arrangements, inadequate measures of display, unreliable sales data, etc. Last but not least, neglecting cross effects—i.e., cross-elasticities—can only further deteriorate direct elasticity measures.[2]

Not surprisingly, most authors involved in optimization-orientated research have built normative models ignoring demand interdependencies. For example, Lynch (1974) maximizes total contribution to profit subject to a space availability constraint and assumes that the item's sales volume is a quadratic function of the space allocated to it. Hansen and Heinsbroek (1979) maximize the total profit generated by all the items to be selected and displayed, subject to total space availability and per item minimum display requirement constraints; they regard space as an integer variable: units of space being identified with facings, and define the sales volume of an item as a constant-elasticity function of the number of facings. Again, cross-elasticities are neglected or at best, supposed to affect indirectly one of the parameters.

Anderson and Amato's (1974) model aims at simultaneously determining both the assortment composition and the allocation of the available display area. Coping only with the direct positive impact of display on the probability of buying the item impulsively, their formulation amounts to a classical "knapsack" problem. They conclude that the optimal assignment should consist in allocating the required minimum number of facings to each of the items selected except for the most profitable one, which is to be assigned all of the display area left available. We show in §4 (Appendix A) that their finding essentially results from their neglecting possible global product class and replenishment cost effects.

In a similar way, the multiple-choice nested knapsack model, advocated for catalog planning by Armstrong, Sinha and Zoltners (1982), could be transposed to solve the shelf space assignment problem by translating the total number of catalog pages into a total number of units of shelf space available, and color (vs. black-and-white) pages in a catalog into eye-level display (vs. bottom-level). Attractive especially because of its high flexibility, such an approach raises implementation issues, caused primarily by the binary nature of the decision variables. In our context, they would be defined as follows: $x_{jk} = 1$, if item k is assigned to the jth class of display (with e.g., $j = 1$ standing for one facing at bottom-level; $j = 2$, standing for one facing at eye-level; etc.); $x_{jk} = 0$, when item k is not assigned to the jth class, or when it is not displayed at all. Then the branch-and-bound algorithm developed would determine in which of the display classes considered each item is to be categorized. This is accomplished through the constrained maximization of the linear combination of items' potential profit contributions. Constant profit contribution coefficients should thus be estimated, one per class-item combination. But of course, this could hardly be done without explicit reference to a predetermined specific allocation plan, at least if one is willing to take into account the interactions between items. In any event, the model user is then left with the task of determining the specification, calibration and discretization of the sales response functions which underlie the definition of the profit contribution coefficients.

As a conclusion of the discussion of the optimization models reviewed so far, we note

[2] With the diffusion of the UPC and computerized scanning, belief in display effectiveness and control strongly develops now. See Owens et al. (1983) and MacZinko (1985).

that interdependencies occur only because of the existence of a total space constraint. As such, sales of different items appear related simply because increasing the space devoted to one of them results in decreasing the space left to the others, given that the total area available is limited.

Two contributions deserve special attention for they explicitly include interactions between items in the model specification. Anderson (1979) deals with a two-brand product line: brand A versus a composite, B, of all other brands. The market share of brand A is modeled as a logistic function of its share of the display space. The product-line profit, including storage costs, is maximized.

The most comprehensive model to date is no doubt Corstjens and Doyle's (1981). Maximizing total store profit within a geometric programming framework, they optimize the space allocation across product categories (width of the assortment). The unique feature of their model lies in the formulation of the demand for each product group. It is defined as a multiplicative, power function of the display areas allocated to all of the product groups. Therefore cross-elasticities explicitly enter into the picture.

This article expands on those authors' pioneering work. A dynamic extension of their original approach can be found in C-D (1983).

3. Theoretical Shelf Space Allocation Model

In this section we examine the following shelf space allocation problem for the profit-maximizing retailer:

$$\underset{s_i}{\text{Maximize}} \left\{ \sum_i g_i q_i(s_1, \ldots, s_i, \ldots, s_n) - \sum_i C_i \right\}$$

$$\text{subject to} \quad \sum_i s_i \le S,$$

$$s_i \ge 0, \qquad \text{for all } i.$$

The problem is therefore analogous to C-D. We do not include additional constraints related to the integer requirements on the number of facings, or control limits imposed by the retailer. Using a math programming algorithm, such constraints can indeed be handled relatively easily. But we prefer recurring to the old-fashioned marginal analysis which is more likely to highlight the substantive economic issues involved. Also rather than having explicit specifications for q_i and C_i, we work with implicit functions, so as to keep the economic interpretation as general as possible.

In order to focus specifically on shelf effects, we define each product's sales volume, q_i, as a nonnegative function of the space allocation only, i.e. the s_j's. Other marketing instruments are therefore assumed to be fixed. Direct effects are positive since product display is expected to boost its sales volume, $\partial q_i/\partial s_i \ge 0$ and $\eta_{ii} \ge 0$. Cross effects are introduced but those may either be negative or positive, depending on the nature of interactions taking place within the assortment under consideration. Substitute items yield negative cross-elasticities, while complementary product classes generate positive ones.

The specification of the cost functions is meant to cover the store product handling operations, i.e. $C_i(q_i, s_i)$, with: $\partial C_i/\partial q_i \ge 0$ and $(\partial C_i/\partial s_i | dq_j = 0, \forall j) \le 0$.

The signs of the first derivatives are almost self-explanatory: higher sales volume as well as reduced (storage) space imply extra replenishment operations.

The Lagrangean function corresponding to the nonlinear programming problem above is defined by:

$$\mathcal{L} = \left(\sum_i g_i q_i - \sum_i C_i\right) + \lambda\left(S - \sum_i s_i\right), \qquad \text{with} \qquad \lambda \ge 0.$$

Assuming that the space-availability constraint is strictly binding, necessary conditions for optimality yield:[3] $\lambda \geq 0$, $\sum_i s_i = S$; and letting $p_i = g_i - (\partial C_i/\partial q_i)$,

$$\sum_i p_i(\partial q_i/\partial s_j) - (\partial C_j/\partial s_j \,|\, dq_j = 0) - \lambda = 0, \qquad \text{for all } j \text{ values.}$$

Solving for s_j, we obtain: $s_j = (\gamma_j C_j + \sum_i \pi_i \eta_{ij})/\lambda$, where $\pi_i = p_i q_i$.

Since $\sum_k s_k = S$, the optimal share of the total available space to be allocated to product j is given by

$$\sigma_j = s_j/S = (\gamma_j c_j + \bar{\eta}_{.j})/(\bar{G} + \bar{N}), \qquad \text{where} \tag{1}$$

γ_j measures the percentage decrease in product j's handling cost resulting from a percentage increase in the space allocated to it;

$c_j = C_j/\Pi$ reflects the relative importance of the replenishment cost associated with carrying item j, as compared to the product line's overall profitability;

$\bar{\eta}_{.j} = \sum_i r_i \eta_{ij}$ is the weighted mean of all elasticities with respect to item j's space share. It captures the percentage overall impact on the product line's profitability of a percentage increase in the space allocated to item j (the higher the profitability of any other item i affected by such a reallocation, the higher the weight attached to the corresponding cross-elasticity, η_{ij}, since: $r_i = \pi_i/\Pi$);

$\bar{N} = \sum_j \bar{\eta}_{.j}$ and $\bar{G} = \sum_k \gamma_k c_k$ are the normalizing terms.

Formula (1) simply implies that priority should be given to products whose display contributes most to boosting the sales of the most profitable ones and to reducing handling costs. It is applicable along the two dimensions—width and depth—delimiting a supermarket's assortment. A pragmatic hierarchical approach would consist of two phases: first along the width-axis, (1) could be used to distribute the total space available *across* the various product classes; second along the depth-axis, it could be transposed for space allocation *within* the lot assigned to each homogeneous product class, i.e. to share it between the substitute items composing that category. In any case, both complementary ($\eta_{ij} > 0$) and substitute products ($\eta_{ij} < 0$) can be processed. On the principles of hierarchical allocation in a marketing context, see, for example, Gijsbrechts (1984) and Gijsbrechts and Naert (1984).

However the optimal allocation formula looks quite different from the common rules of thumb reviewed in the preceding section. Rather restrictive conditions should indeed be imposed to derive them as special cases of (1); hence, their validity appears quite limited. For example, if altering product display does not influence sales (i.e., $\eta_{ij} = \bar{\eta}_{.j} = \bar{N} = 0$), equation (1) boils down to $\sigma_j = \gamma_j C_j/\sum_i \gamma_i C_i$. If in addition we regard handling costs as constant-elasticity functions of the replenishment operations (reflected in sales per unit of space),

$$C_j = v_j(q_j/s_j)^{\gamma}, \tag{2}$$

where $v_j > 0$ is a scaling factor linked to product j's facing volume,[4] then it can be shown that

$$\sigma_j = v_j^{e_1} m_j^{e_2} / \sum_i v_i^{e_1} m_i^{e_2}, \tag{3}$$

with: $e_1 = 1/(1 + \gamma)$ and $e_2 = \gamma/(1 + \gamma)$.

The PROGALI rule, $\sigma_j = m_j$, matches the latter expression when the cost-elasticity, γ, is relatively high (because then $e_1 \cong 0$ and $e_2 \cong 1$). Thus only in a few very special cases might it lead to an allocation close to optimality. Connections with alternative rules of thumb are explored in the next section.

[3] If function q_i is concave, these conditions are also sufficient.

[4] Note that we can e.g., let $v_j = c_j u_j^{\gamma}$, with u_j representing the space taken up by a unit of product j, and c_j, the cost per complete replenishment operation (i.e., $s_j = q_j u_j$).

4. SH.A.R.P.: A Realistic Specification

System (1) is highly nonlinear since *all* terms on the right-hand side will in general be nonlinear functions of the space shares, σ_j. Even the over-simplification required to recover the PROGALI rule does not necessarily lead to closed form expressions: the nature of (3) will depend on the specification, chosen for the sales shares, m_i. This notwithstanding, numerical heuristics can be devised to solve (1) and determine satisfactory—if not optimal—allocations.

The feasibility of such an approach is illustrated hereafter in the case of *a* product class composed of competitive brands offered in different variety types (e.g., package sizes). An appropriate items' sales model is made explicit and substituted into (1) to arrive at a workable allocation formula, i.e. the SH.A.R.P. rule. Next we examine under what conditions SH.A.R.P. may end at rules of thumb. An estimation method, as well as an iterative calculation procedure, are then suggested and tested.

Operationalizing the Optimal Allocation

While C-D focus on assortment width and interactions between product classes, here we deal with *assortment depth and interactions between substitute items within a product class.* Accordingly, to model interdependencies we break down sales elasticities into their product-class level and item sales share components,

$$\eta_{ij} = \epsilon_j + \mu_{ij}. \tag{4}$$

Implementation of (1) further requires the explicit specification of the sales response function. Substitution effects can conveniently be represented by the attraction model, advocated on various grounds by many marketing scholars (see, e.g. Bultez and Naert 1975, Little, Bell and Keeney 1975, Nakanishi and Cooper 1973, Weverbergh, Naert and Bultez 1981). Applied to item share, it takes the following form

$$m_i = \alpha_i s_i^{\beta_i} / \sum_{c=1}^{n} \alpha_c s_c^{\beta_c}, \tag{5}$$

where the α_i- and β_i-parameters are positive constants. It corresponds to a flexible multivariate logistic relationship. Depending on the value of β_i, the share, m_i, will be either an *S*-shaped (for $\beta_i > 1$) or a strictly concave ($0 < \beta_i < 1$) function of s_i—at least, ceteris paribus.[5] Specification (5) is robust: if an item is not displayed ($s_i = 0$), its share of the assortment sales is equal to zero; if, on the other hand, the space allocated to an item increases, its share tends asymptotically to one.

Since shelf space is the only merchandising variable examined here, the α_i coefficient sums up the effects of the other variables in the marketing mix. It is supposed to measure the intrinsic preference for the corresponding item: i.e., the attraction exerted by item *i*, independently from the effect produced by its visibility. So when the latter is neutralized—e.g., when $\beta_j = \beta$ and $s_j = S/n$, for all *j*—the sales share distribution is determined by $m_i = \alpha_i / \sum_c \alpha_c$.

Special displays and promotions are excluded but whenever they occur their impact can be inserted in the model without much difficulty. For convenience of notation error terms are omitted.

The model leads to the following elasticities:

$$\mu_{ij} = \beta_j(\delta_{ij} - m_j), \qquad \forall i, j, \tag{6}$$

where δ_{ij} is a Kronecker delta, i.e. $\delta_{ij} = 1$, for $j = i$, and $\delta_{ij} = 0$, otherwise. They were first derived by Bultez (1975, pp. 189–207), who also indicated some restrictive properties of

[5] That is when the function variation is not bound by the space-availability constraint.

(5): constant cross elasticities (i.e. $\mu_{ij} = \mu_{kj}$, $\forall i, k \neq j$), linearity of their variations with items sales shares, etc. This specification can of course be generalized in several ways (see e.g., Wagner and Taudes 1986) but for our purpose it offers enough flexibility, as proved in Figure 1.

In addition, as equation (6) and Figure 2 reveal, the direct elasticity of the item sales share, μ_{jj}, decreases with the level of α_j. This feature concretely expresses a reasonable assumption regarding the shopper's behavior: i.e., the stronger his preference for the item (reflected in α_j), the lower the impact of the item visibility on his choice; on the contrary, for items bought on impulse—the sales of which critically depend on their visibility (the shopper's choice being unplanned)—low values for the α_j's are to be expected.

Substituting (4) and (6) into $\bar{\eta}_{\cdot j}$, we obtain:

$$\bar{\eta}_{\cdot j} = \sum_i r_i[\epsilon_j + \beta_j(\delta_{ij} - m_j)],$$

and since $\sum_i r_i = 1$, $\bar{\eta}_{\cdot j} = \epsilon_j + \beta_j(r_j - m_j)$. Incorporating this expression in the allocation formula (1) yields

$$\sigma_j = \epsilon_j^* + c_j^* + \beta_j^*(r_j - m_j), \qquad \text{with:} \tag{7}$$

$$\epsilon_j^* = \epsilon_j/(\bar{N} + \bar{G}),\ c_j^* = \gamma_j c_j/(\bar{N} + \bar{G}) \qquad \text{and} \qquad \beta_j^* = \beta_j/(\bar{N} + \bar{G}).$$

As such the SH.A.R.P. rule appears nontrivial to apply since all of its terms depend themselves on space allocation. A computer heuristic is therefore required to solve the system of nonlinear equations defined by (7). We suggest a fairly simple one and test its performance in the subsection dealing with the calculation of the optimum.

Studying specific extreme cases may also be instructive. So in Appendix A we show how Anderson and Amato's (1974) result can be derived from (7).

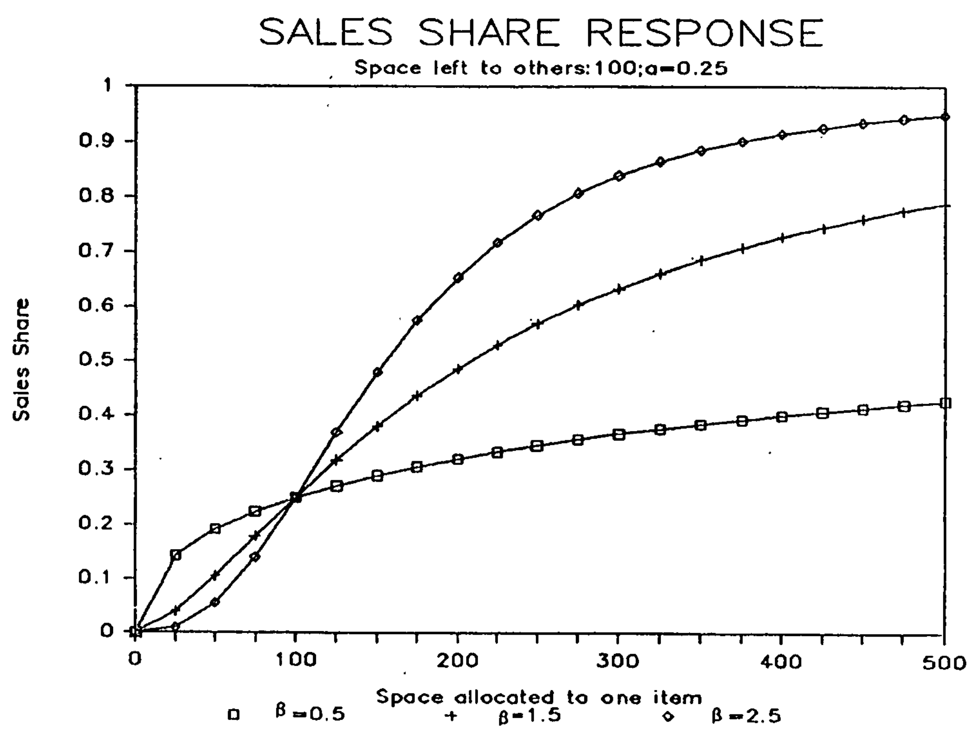

FIGURE 1

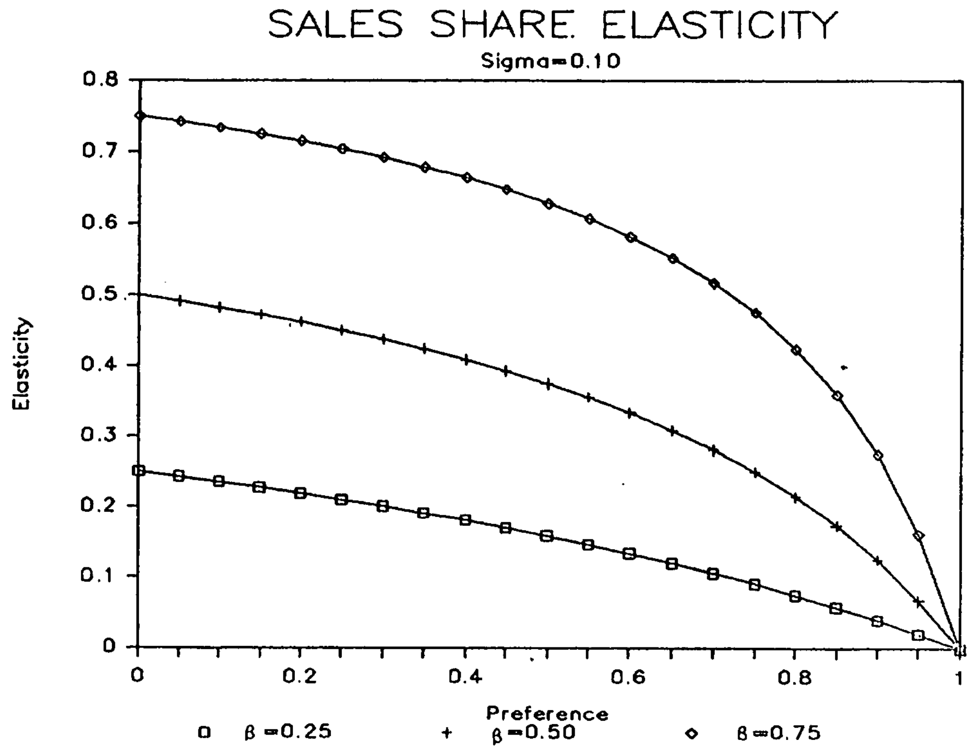

FIGURE 2

Confronting SH.A.R.P. with Current Practice

Linking rules of thumb to the result of our analysis commands the introduction of a few simplifying assumptions. First, consider that the incidence of the shelf reallocation on the assortment total sales volume (Q) is negligible, then $\epsilon_j \cong 0$ for all j. This case may indeed be met quite often in practice, especially if as we postulate it, the total space used to display the assortment is fixed (S) and if the assortment composition cannot be altered.

Second, empirical investigations have shown that letting $\beta_j = \beta$ for all j does not cause much damage to fits and forecasts produced by the attraction model. For supporting evidence, see Bultez and Naert (1975) and Naert and Weverbergh (1981). Brodie and de Kluyver (1984) and Ghosh et al. (1984) present examples to the contrary, but Naert and Weverbergh (1984) argue that even in those instances, imposing $\beta_j = \beta$ may be desirable.

Third, replenishment costs can be modeled by a simplified variant of (2), i.e. $C_j = v(q_j/s_j)$, provided items package sizes and designs do not differ greatly. Although the latter assumption may be questionable, charging different handling costs to the various items appears an arduous accounting exercise. The current upsurge of interest for the so-called DPP systems clearly confirms that direct costing of supermarkets' operations remains a desirable objective; see e.g., Pinnock (1986).

Integrating the assumptions discussed hereabove into (7), gives

$$\sigma_j = \frac{m_j/s_j}{\Sigma_i\,(m_i/s_i)} + \frac{\beta}{\Sigma_i\,c_i}(r_j - m_j), \tag{8}$$

for $\bar{N} = \Sigma_j\,\beta(r_j - m_j) = 0$.

As will be numerically confirmed hereafter, a good approximation of the optimum may be obtained if we apply the allocation formula once, on the grounds that the space available has been equally distributed between the items (i.e. $s_j^{(0)} = S/n$ for all j). Then (8) becomes

$$\sigma_j^{(1)} = \beta^{*(0)} r_j^{(0)} + (1 - \beta^{*(0)}) m_j^{(0)}, \tag{9}$$

where: $\sigma_j^{(1)}$ stands for the allocation obtained as a first approximation to (8),

$$m_j^{(0)} = \alpha_j / \sum_i \alpha_i, \qquad p_j^{(0)} = g_j - (vn/S),$$

$$r_j^{(0)} = \alpha_j p_j^{(0)} / \sum_i \alpha_i p_i^{(0)}, \qquad c_j^{(0)} = (vn/S)\alpha_j / \sum_i \alpha_i p_i^{(0)}, \qquad \text{and}$$

$$\beta^{*(0)} = \beta / \sum_i c_i^{(0)}.$$

Relationship (9) demonstrates that the "truth" lies somewhere between the two extreme benchmarks used by merchandisers: the CIFRINO/OBM vs. the PROGALI rules. Given that handling costs are likely to account for a large part of a supermarket's total labor cost (which itself may amount to 15% of its sales revenue) and given that we expect shelf-space elasticities to be significantly lower than one, conservative values of the β^* coefficient will frequently be confined to the [0, 1]-range. Hence, *our norm* for σ_j, as re-expressed in (9), *defines each item's space share as a weighted mean of its relative contribution to the assortment profitability* (r_j) *and of its share of the assortment total sales volume* (m_j). The weighting factor, β^*, is determined by the ratio of the impact of items' visibility on sales (as measured by β) to the relative incidence of handling costs on the assortment profit ($\sum_i c_i = \sum_i C_i/\Pi$). Therefore, for product categories demanding huge manipulations ($\beta^* \cong 0$), an allocation based on relative sales volumes will probably be close to optimal; while for lines of products bought essentially on impulse ($\beta^* \cong 1$), an allocation based on profit contributions is recommended.

An *internal consistency test* can also be obtained as a by-product of equation (8). It may serve as a control tool to check actual space assignments. Provided all the assumptions underlying (8) are acceptable and provided supermarket managers allocate space quasi-optimally, regression analysis of cross-sectional data on item's space, profit and sales shares should determine the relative weight (implicitly) attached by merchandisers to the two essential determinants of the optimal allocation. Transforming (8) into the following regression equation

$$(\sigma_j - \varphi_j) = \beta^*(r_j - m_j), \qquad \text{with:} \qquad \varphi_j = (m_j/s_j) / \sum_i (m_i/s_i),$$

one realizes that the least-square estimate of β^* is defined by

$$\hat{\beta}^* = \sum_j [(\sigma_j - \varphi_j)(r_j - m_j)] / \sum_j (r_j - m_j)^2. \tag{10}$$

The observed goodness-of-fit, as measured by the following correlation coefficient between the fitted space shares, $\hat{\sigma}_j$, and the actual ones, σ_j,

$$\rho = \hat{\beta}^* [\sum_j (r_j - m_j)^2 / \sum_j (\sigma_j - \varphi_j)^2]^{1/2},$$

will be an indication of both the model validity and the merchandiser's expertise. Given that lack of fit may possibly be due to a flaw in the approximation of the optimal allocation defined by (8), we advise against using $\hat{\sigma}_j$ as the absolute norm automatically dictating the space reallocation. Instead, we would suggest regarding this test as a problem-finding device: residuals produced by such a regression analysis point the merchandiser's attention at product items, the display of which may be questionable.

Optimal reallocation should rather be based on direct estimates of the parameters of model (5) because even if the current space allocation looks consistent on the average, the merchandiser's implicit subjective estimates of preference (α_i) and substitution (β_i) parameters may well be consistently biased. Therefore we present in the following subsection the appropriate calibration and test procedures.

Econometric Measurement of the Response Parameters

Linearization procedures developed to estimate the parameters of the attraction model have been discussed extensively in the literature on market share models (Nakanishi and Cooper 1974, Bultez and Naert 1975, Bultez 1978). Referring to its simplest variant leading to the linear equation derived above, i.e. (5) with $\beta_i = \beta$ for all i values, the least-square estimates of its parameters can be obtained on the basis of the following regression equation:

$$y_{it} = \sum_{c=1}^{n-1} a_c d_{it,c} + \beta x_{it}, \qquad \text{with:} \tag{11}$$

$$a_c = \ln \alpha_c, \quad \text{for} \quad c = 1, 2, \ldots, n-1; \quad \alpha_n = 1;$$

$$y_{it} = \ln m_{it} - \ln m_{nt}; \quad y_{nt} = (n-1) \ln m_{nt} - \sum_{i=1}^{n-1} \ln m_{it};$$

$$d_{it,c} = +1, \quad \text{for} \quad i = c \quad \text{and} \quad d_{it,c} = 0, \quad \text{otherwise};$$

$$d_{nt,c} = -1, \quad \text{for all} \quad c \neq n;$$

$$x_{it} = \ln s_{it} - \ln s_{nt}; \quad x_{nt} = (n-1) \ln s_{nt} - \sum_{i=1}^{n-1} \ln s_{it}.$$

The t-subscript identifies repeated observations made in the course of a *controlled experiment*: $t = 1, 2, \ldots, T$; i refers to product items.

Our discussion of usual allocation habits and of rule (9) shows that an item's space share will most often be a function of its share of the assortment total sales volume. Therefore estimators derived from cross-sections of historical data on past allocations are likely to be biased by the resulting simultaneous dependence. Hence observations should be collected within the course of an experiment during which space is reallocated in various ways deviating markedly from current practice.

To validate (5), we used the data generated by such an experiment conducted in the *Albert Heijn*'s Dutch retail chain. It extended over a 24-week period and was run in six supermarkets. Estimates of β derived from the application of the OLS method to equation (11), are reported in Table 2, for the *milk-drink product category*. Five major brands were treated: *Nutroma, Friesevlag, Halvamel, Elcemel and Becel*; other brands with small market shares were grouped and dealt with as a composite sixth alternative.

Price promotion and three dummy variables accounting for differences in shelf height were also included but here we limit our comments to the impact of modifications in shelf space. The 95%-confidence intervals for β and values of Student's statistic (in parentheses) corresponding to the null hypothesis, $\beta = 0$, are also displayed in Table 2. Obtained with this large sample ($n = 6$ and $T = 24$), they clearly establish the significance of the substitution effects resulting from variations in brands visibility.

However, in view of the span of the confidence intervals, one may wonder if the estimates are sufficiently precise for allocation purpose. This question is addressed in the sensitivity analysis subsection.

TABLE 2
OLS-Estimation of the Intensity of Substitution Effects Generated by the Display-Area Allocation

Supermarket	$\hat{\beta}$	R^2	95% Confidence Interval for β
1	0.460 (6.501)	0.738	[0.320–0.600]
2	0.458 (3.376)	0.713	[0.189–0.746]
3	0.455 (8.350)	0.608	[0.347–0.563]
4	0.307 (7.622)	0.850	[0.227–0.387]
5	0.298 (4.870)	0.720	[0.177–0.420]
6	0.276 (3.353)	0.718	[0.113–0.438]

Improving Shelf Space Allocation

Although equation (8) looks simpler, its degree of nonlinearity is not lower than it is for (7). Therefore, and also because the latter expression is subject to less restrictive assumptions than (8), we continue to refer to it and suggest the following simple heuristic designed to solve (7).

Step # 0. Define the initial allocation by giving all items equal space, i.e. $\sigma_j^{(0)} = 1/n$; then the resulting sales shares are essentially determined by the preference parameters, i.e. $m_j^{(0)} = \alpha_j(S/n)^{\beta_j}/\Sigma_c\, \alpha_c(S/n)^{\beta_c}$. Compute the corresponding values[6] of $\epsilon_j^{*(0)}$, $c_j^{*(0)}$, $\beta_j^{*(0)}$ and $p_j^{(0)}$. Set $k = 1$.

Step # k. Revise the previously determined allocation—at step # $(k-1)$—according to (7), i.e.

$$\sigma_j^{(k)} = \epsilon_j^{*(k-1)} + c_j^{*(k-1)} + \beta_j^{*(k-1)}[r_j^{(k-1)} - m_j^{(k-1)}], \qquad \text{where:} \tag{12}$$

$$m_j^{(k-1)} = A_j^{(k-1)}/\sum_c A_c^{(k-1)}, \qquad r_j^{(k-1)} = p_j^{(k-1)}A_j^{(k-1)}/\sum_c p_c^{(k-1)}A_c^{(k-1)}, \qquad \text{with:}$$

$$A_j^{(k-1)} = \alpha_j[s_j^{(k-1)}]^{\beta_j} \quad \text{and} \quad s_j^{(k-1)} = \sigma_j^{(k-1)}S \quad \text{for} \quad j = 1, 2, \ldots, c, \ldots, n.$$

Test the degree of variation from one iteration to the next; search for $\mathrm{MAX}_j\,\{|\sigma_j^{(k)} - \sigma_j^{(k-1)}|/\sigma_j^{(k)}\} = V^{(k)}$. When $V^{(k)}$ becomes negligible—i.e. $V^{(k)} \leq \delta$, an arbitrary small positive number—then stop. Otherwise set $k = k + 1$ and continue revising the allocation, applying (12) iteratively.[7]

Vouching for the convergence of this computational procedure and proving that when convergence occurs the recommended space allocation is globally optimal goes

[6] Superscripts indentify the iteration numbers. Unless product-class sales elasticities are constant and replenishment costs are negligible, all those determinants of the optimal space allocation will also vary with the current allocation itself, i.e. $\sigma_j^{(k-1)}$. Therefore they should also be superscripted but as they depend on the specific forms given to the overall product class sales response and to the replenishment cost functions, we refrain from making their expression explicit.

[7] In degenerate cases—which might occur when the product assortment has been poorly selected—(13) might yield a negative value; therefore a nonnegativity test is also performed to detect such anomalies.

beyond the scope of this article. Nevertheless, we pretested this heuristic over a wide range of parameter values for the sales share response model (i.e., the α_j and β_j's) and of realistic levels for gross margins (the g_i's). A constant-elasticity function was postulated for the assortment total sales volume, i.e.

$$Q = Q_0 \cdot (\prod_{i=1}^{n} s_i)^{\epsilon}, \tag{13}$$

and linear functions used for replenishment costs, i.e. (2) with $\gamma = 1$. Our numerical experience indicates that although the heuristic is based on first-order conditions and not on the Hessian matrix, it generally converges rapidly. We observed that divergence might happen when the β_j^*'s get larger than unity or when gross margins vary considerably across items. In those situations, using the equi-repartition as the initial allocation (i.e. $\sigma_j^{(0)}$) reveals inadequate but alternative initial allocations have been tried out successfully: e.g., we simply used the solution obtained for lower values of the β_j's (or for higher values of the v_j's) as a starting point.

To assess the validity of the allocations arrived at, we systematically scanned the profit functions for assortments made up of only 2 items (the case considered by Anderson 1979); then σ_1 is the only independent variable, since $\sigma_2 = 1 - \sigma_1$. For larger assortments (up to $n = 20$ items), we randomly explored the profit functions and also had the initial allocation changed to check the robustness and stability of the solution. Under no circumstances could we find an alternative solution yielding a profit higher than the (maximum) one determined by SH.A.R.P. heuristic.

For illustration purposes, Table 3 sums up the results obtained when the procedure is applied to allocate the shelf-space available for the milk-drink assortment in the first Albert Heijn's supermarket. Replenishment costs are not taken into account and a relatively low elasticity is assumed for the assortment total sales ($\epsilon \leq 0.10$). Three values are considered for the key-parameter, β, the least-square point estimate, 0.46, and the limits of its 95%-confidence interval, i.e. 0.32 and 0.60 (see 1st row in Table 2). The α_i are derived from the least-square estimates of the a_i and normalized, i.e. $\alpha_i = e^{\hat{a}_i}/(1 + \sum_{c=1}^{n-1} e^{\hat{a}_c})$. The criterion used for the stopping rule, δ, is set equal to 10^{-5}.

Speed of convergence can be appreciated by the closeness of the results from the 1st iteration to those obtained at the outcome. The largest discrepancy, observed for brand 3 and $\beta = 0.60$, hardly reaches 2.5%: i.e., (0.167 − 0.163)/0.163.

TABLE 3

Allocation Derived from SH.A.R.P. Heuristic

Parameters: $\alpha_1 = 0.18$, $g_1 = 2.75$; $\alpha_2 = 0.27$, $g_2 = 2.70$; $\alpha_3 = 0.22$, $g_3 = 2.30$; $\alpha_4 = 0.06$, $g_4 = 1.15$; $\alpha_5 = 0.06$, $g_5 = 1.30$; $\alpha_6 = 0.21$, $g_6 = 2.00$; $Q_0 = 100$ $S = 10{,}000$ $\epsilon = 0.10$

	$\beta = 0.32$			$\beta = 0.46$			$\beta = 0.60$		
	1st Iteration	Final Iteration		1st Iteration	Final Iteration		1st Iteration	Final Iteration	
i	$\sigma_i^{(1)}$	$\sigma_i^{(F)}$	$m_i^{(F)}$	$\sigma_i^{(1)}$	$\sigma_i^{(F)}$	$m_i^{(F)}$	$\sigma_i^{(1)}$	$\sigma_i^{(F)}$	$m_i^{(F)}$
1	0.1856	0.1854	0.1846	0.1939	0.1933	0.1893	0.2022	0.2008	0.1956
2	0.1919	0.1919	0.2799	0.2030	0.2027	0.2903	0.2140	0.2133	0.3042
3	0.1668	0.1662	0.2178	0.1669	0.1650	0.2152	0.1670	0.1630	0.2109
4	0.1507	0.1512	0.0576	0.1437	0.1452	0.0553	0.1367	0.1400	0.0525
5	0.1528	0.1532	0.0579	0.1467	0.1478	0.0558	0.1406	0.1430	0.0532
6	0.1522	0.1522	0.2022	0.1458	0.1459	0.1941	0.1395	0.1399	0.1837

TABLE 4
Comparison with Various Suboptimal Allocations

Degree of Suboptimization Resulting from the Use of Alternative Rules						
Parameters / Rules	$\epsilon = 0.05$, $\beta = 0.32$	$\epsilon = 0.05$, $\beta = 0.46$	$\epsilon = 0.05$, $\beta = 0.60$	$\epsilon = 0.10$, $\beta = 0.32$	$\epsilon = 0.10$, $\beta = 0.46$	$\epsilon = 0.10$, $\beta = 0.60$
First Iteration:						
SH.A.R.P	0.0017%	0.0127%	0.0609%	0.0002%	0.0016%	0.0074%
PROGALI	3.287%	3.131%	3.211%	7.634%	7.237%	7.014%
OBM	7.094%	6.626%	6.463%	15.836%	15.185%	14.762%
Steady State:						
PROGALI	7.032%	10.270%	16.857%	15.310%	21.644%	33.105%
OBM	13.971%	19.328%	29.189%	28.653%	38.034%	52.755%

In Table 4, we compare the maximum profit (Π_S) generated by the allocation determined by $\sigma_i^{(F)}$, the final iteration of SH.A.R.P. heuristic, to the levels (Π_A) reached when relying upon alternative rules. The performance of SH.A.R.P. can best be evaluated by looking at the percentage decrease in profitability caused by the choice of any alternative, i.e. $[(\Pi_S - \Pi_A)/\Pi_A] \times 100\%$.

Deviation of SH.A.R.P. 1st iteration profit from the maximum, Π_S, is also provided as a benchmark. It appears negligible, which confirms the swiftness of the heuristic. Of more importance, Table 4 makes clear to what extent SH.A.R.P. may outclass usual rules of thumb, such as OBM and PROGALI.

Starting from an equal repartition of the available space (as for SH.A.R.P. heuristic) and applying the PROGALI rule yields for the 1st round, $\sigma_i^{(1)} = g_i\alpha_i / \sum_{c=1}^{n} g_c\alpha_c$. Successive recourse to the same rule ultimately leads to the following steady state allocation,

$$\sigma_i^{(F)} = [g_i\alpha_i]^{1/(1-\beta)} / \sum_{c=1}^{n} [g_c\alpha_c]^{1/(1-\beta)},$$

as proved in Appendix B. Corresponding expressions for the OBM rule are obtained by dropping out the g_i coefficients, since the latter allocates space proportionately to sales instead of allotting it in direct ratio with gross margins.

In the case examined here—i.e., assuming product visibility has a significant impact on their sales and that replenishment costs are negligible—both sets of rules of thumb yield profits that are substantially below the maximum reached with SH.A.R.P. Moreover, Table 4 reveals that consistent persistence in the implementation of PROGALI and OBM produces (steady state) outcomes that are much worse than those obtained when limiting it to their myopic application (first iteration results). In view of our discussion of equation (9) and related practice, this finding should cause no big surprise.

Potential gains to be expected from giving SH.A.R.P. preference over alternative rules appear undeniable, but a fair comparison would require running in-store experiments designed for dealing with the various allocation rules as treatments. This would secure our measurements against model-specific biases.[8] Although we have not yet been offered the opportunity to run such an elaborate test, the actual benefits obtained so far owing to the implementation of SH.A.R.P. keep the promise of Table 4. Parts of those benefits are commented upon in the following sections.

[8] Of course, the dependability of our comparative evaluation varies as the validity of the mathematical specification underlying SH.A.R.P. For a discussion of this issue see e.g., Naert and Leeflang (1978, pp. 359–62).

TABLE 5

Impact of Errors in the Measurement of Substitution Effects

Assumed Estimated Values \ Assumed Parameter Values	$\epsilon = 0.05$			$\epsilon = 0.10$		
	$\beta = 0.32$	$\beta = 0.46$	$\beta = 0.60$	$\beta = 0.32$	$\beta = 0.46$	$\beta = 0.60$
$\hat{\beta} = 0.32$	—	0.089%	0.331%	—	0.044%	0.187%
$\hat{\beta} = 0.46$	0.086%	—	0.073%	0.055%	—	0.051%
$\hat{\beta} = 0.60$	0.307%	0.071%	—	0.186%	0.045%	—

Sensitivity Analysis to Estimation Inaccuracy

When evaluating the estimates displayed in Table 2, our attention was called to the span of the confidence intervals obtained for the key-parameter, β. Table 5 sets our minds at ease on that point.

Entries in that table give the percentage differences between the maximum profit computed on the basis of the (assumed) real parameter value ($\Pi_S(\beta)$) and the profit level reached when the allocation is based on the (assumed) estimated value ($\Pi_S(\hat{\beta})$). They appear low enough for us to be satisfied with the precision of the β-estimate; however, this low sensitivity is partly due to the fact that the measurement error is assumed to have the same size and direction for all items, since we let $\beta_i = \beta$, for all i values.

5. Implementing SH.A.R.P.

Over the 1984–1987 period, we could proceed to several tests of our methodology on various assortments (i.e. coffee, biscuits, canned fruits and pet-foods) in four different Belgian supermarket chains (namely, CASH-BATTARD, CHOC-DISCOUNT, NOPRI and UNIDIS). Reallocations guided by the implementation of SH.A.R.P. led to actual increases in the profitability varying from a low of 6.9% to a high of 33.8% and this without affecting handling and replenishment operations (see e.g. P. Battard and A. Bultez 1985).

By way of example, the first experiment conducted on the dog-food assortment of a CASH-BATTARD store is described hereafter. A variant of allocation rule (7)—adapted to the CASH-BATTARD situation—determined the changes to be treated. It was first parameterized on the basis of a preliminary analysis of the allocation prevailing before the experiment was run. The space modification tested was dictated by an approximation of the optimization rule (1st iteration). Resulting variations in items' sales shares were then related to changes in space allocation and so objective estimates of the key-parameters were obtained. The actual increase in profit was eventually compared to the expected one. The next subsection goes into all the details of those operations. Capitalizing on our experience we conclude by suggesting a sequential approach to SH.A.R.P. implementation.

The CASH-BATTARD Success Story

In a preliminary stage, we analyzed how the display space available in that store was usually allocated, referring to the type of internal consistency check discussed in relation with (8) and (9). Total assortment sales being regarded as rather insensitive to reallocations ($\epsilon_j \cong 0$), handling costs being supposed to vary linearly ($\gamma_j \cong 1$), it was further assumed that differences between items replenishment costs could safely be

TABLE 6

Data Relevant to the Cash-Battard Experiment

Product	Number of Items		Equal Allocation: Before Experiment 24 Weeks	Equal Allocation: Experiment Phase 1	Re-allocation Experiment Phase 2
CHAPPI	2	Space share	0.3333	0.3333	0.2334
		Sales share	0.3405	0.3022	0.2174
		Profit share	0.2808	0.2428	0.1611
PAL	4	Space share	0.3333	0.3333	0.5000
		Sales share	0.3747	0.3855	0.5650
		Profit share	0.5156	0.5174	0.6979
PLUTO	2	Space share	0.3333	0.3333	0.2666
		Sales share	0.2848	0.3123	0.2176
		Profit share	0.2036	0.2398	0.1410

information system which is not yet available. Thus in that case, (7) reduces to,[9]

$$\sigma_j = (1/n) + \beta^*(r_j - m_j), \tag{14}$$

since then: $\bar{N} = 0$, $\bar{G} = \sum_k c_k$ and $c_j^* = c_j/\sum_k c_k \cong 1/n$.

The realism of this specification can be evaluated by the quality of its fit to the data made available by the supermarket before the experiment was designed. Observations on the previous year sales volumes, margins and facings were gathered for the major items carried by the store in its dog-food line (i.e., 13 items all displayed at the eye-level position). Application of ordinary least squares to (14) gave on that sample,

$$\hat{\beta}^* = \sum_j (r_j - m_j)\sigma_j / \sum_j (r_j - m_j)^2 = 1.092,$$

with an associated t-value, 2.224, and a 0.54 correlation between actual and fitted space. Some of the residuals, $(\sigma_j - \hat{\sigma}_j)$, were large and discussions with the store manager indicated that fear of stockouts was the only rationale for the excessive "piling" of some items on the shelf. Those results pushed the management of the supermarket chain to undertake an experiment with SH.A.R.P.

Given that five of the items correspond to the retailer's own private label, a quasi-experiment was set up relative to the remaining 8 items representing the three dominating nationally advertised brands: CHAPPI, PAL and PLUTO. Those brands had been given equal shelf space for all of the past 24 weeks, which enabled us to get "prior" estimates for the α_i's, on the basis of average weekly sales. As shown in Table 6, sales shares in the period before the experiment were 0.3405, 0.3747 and 0.2848, respectively. With the leading brand, PAL, as reference,[10] i.e. assuming $\alpha_2 = 1$, we obtain: $\hat{\alpha}_1 = 0.3405/0.3747 = 0.909$ and $\hat{\alpha}_3 = 0.2848/0.3747 = 0.760$.

During the first phase of the experiment, we continued the same allocation, thus maintaining the equi-repartition over another 6-week period, but carefully controlling the stocks, price changes, and promotional activities. Comparisons of the before-exper-

[9] Equation (14) can also directly be derived from (8) if one can assume that current allocation is proportional to sales, i.e. $s_j = m_j S$. Note that from an econometric point of view, (14) is a naturally sum-constrained model, i.e. $\sum_j \hat{\sigma}_j = \sum_j \sigma_j = 1$. For a discussion of related issues see Weverbergh, Naert and Bultez (1981).

[10] As $s_i = S/n$ and $\beta_i = \beta$ for all i values, $m_i = \alpha_i/\sum_c \alpha_c$. Since the α_i's are unique up to a multiplicative constant, we can set $\alpha_2 = 1$, e.g., and $m_i/m_2 = \alpha_i/\alpha_2 = \alpha_i$. We can also let $\sum_c \alpha_c = 1$, in which case $m_i = \alpha_i$.

iment with the 1st-period data, displayed in Table 6, revealed no significant changes in the relative performance of those brands. Hence we pooled both data sets together.

The second phase consisted in testing a SH.A.R.P. based allocation for a second 6-week period. In the absence of a classical econometric estimate for β (i.e. derived from a regression analysis of (11)), we took the risk of relying on $\hat{\beta}^* = 1.092$, which as previously pointed out (see comments related to (8) and (9)), may be biased. So the space re-allocation was guided by (12) but constrained by feasibility aspects. Using the data reported in Table 6, we can verify that for PAL, e.g., the first iteration with $\hat{\beta}^*$ yields

$$\sigma_2^{(1)} = (1/3) + 1.092(0.5156 - 0.3747) = 0.487.$$

For practical reasons, PAL actually received half the available space and the remaining was divided out amongst the two other brands according to similar considerations.

Conventional econometric estimates of the α_i's and β were then derived by fitting equation (11) to the data collected. They can be found in Table 7.

A constrained version of (11) was also tested, fixing the α_i's at their "prior" values, i.e. those consistent with the relative sales volumes observed just before the experiment started ($\hat{\alpha}_i$). Thus β was also estimated on the basis of the following regression equation, $[y_{it} - \sum_{c\neq 2} \hat{a}_c d_{it,c}] = \beta x_{it}$, with $\hat{a}_1 = \ln 0.909 = -0.096$, $a_2 = \ln 1 = 0$, and $\hat{a}_3 = \ln 0.760 = -0.274$.

Given that the relative sales levels recorded during the first period of the experiment did not differ much from those observed just before, β-estimates are pretty close. The value obtained and the associated t-statistic reflect the huge influence product visibility exerts on the purchaser (who is not the consumer, hopefully!) of dog food.

The face validity of our estimates of β and β^* was subsequently assessed, before activating the SH.A.R.P. heuristic. According to CASH-BATTARD management, handling costs amount approximately to 13.8% of their supermarket revenue (i.e., $\sum_i C_i \cong 0.138SR$); taking into account their adjusted 18.5% margin on the pet-food assortment (i.e., $\Pi \cong 0.185SR$), the relative cost of the replenishment operations is believed to be $\sum_i c_i = 0.138/0.185 = 0.746$. Adjusted accordingly the β^* value derived from the preliminary review of past allocation routine, we see that the value implicitly attached by the store manager to the key-parameter is given by: $\hat{\beta} = \hat{\beta}^*(\sum_i c_i) = 1.092 \times 0.746 = 0.815$. It appears perfectly consistent with the unconstrained estimate, derived from the regression analysis of the experimentation results.

Confidence in these estimates induced us to use SH.A.R.P. heuristic to determine the best allocation possible. Using $\hat{\beta} = 0.839$, $\hat{\alpha}_1 = e^{-0.171} = 0.843$, $\alpha_2 = 1$ and $\hat{\alpha}_3 = e^{-0.292} = 0.747$, our heuristic leads to the allocation defined in Table 8. Once more the 1st iteration results are quite close to the ultimate ones. Table 9 illustrates the nonnegligible

TABLE 7

Estimates of the Parameters of the Sales Share Attraction Model

	$\hat{a}_1$	$\hat{a}_2$	$\hat{a}_3$	$\hat{\beta}$	Degrees of Freedom
Unconstrained:					
Estimate	−0.171	0	−0.292	0.839	3
t-statistic	(−2.868)		(−5.234)	(13.689)	
Constrained:					
Estimate	−0.096	0	−0.274	0.895	5
t-statistic				(10.778)	

TABLE 8

SH.A.R.P. Based Allocation in the Cash-Battard Case

Product	SH.A.R.P. First Iteration	SH.A.R.P. Final Iteration	
i	$\sigma_i^{(1)}$	$\sigma_i^{(F)}$	$m_i^{(F)}$
CHAPPI	0.2681	0.2629	0.2697
PAL	0.4872	0.4846	0.5128
PLUTO	0.2447	0.2525	0.2175

betterment which can be expected from the implementation of a shelf space allocation inspired by SH.A.R.P. The real results, reached at the term of the second 6-week period of experimentation, revealed to be even more satisfying: profit had actually increased by 11.8%, against the 9.25% predicted by the calibrated model.

Suggested Model Testing Approach

The encouraging results obtained all through the various experiments authorize us to recommend the parsimonious joint measurement-allocation process described in Table 10.

Although it constitutes a mix of our stepwise heuristic with the sequential quasi-experimental design followed in the CASH-BATTARD case (note that iteration superscripts become period subscripts), it sharply distinguishes the estimation of the preference parameters (α_i) from the estimation of the visibility impact parameter (β). Thereby it simplifies the estimate of the latter. Thus as $s_{j1} = S/n$, $m_{j2}/m_{j1} = s_{j2}^{\beta}/[\sum_c \alpha_c s_{c2}^{\beta}]$ and after transforming into logarithms, $[\ln m_{j2} - \ln m_{j1}] = \beta_0 + \beta \ln s_{j2}$, with: $\beta_0 = -\ln[\sum_c \alpha_c s_{c2}^{\beta}]$. Hence, $\hat{\beta}$ is the least-square estimate of the slope in that regression equation; it is defined accordingly in Table 10.

Our methodology can easily be adapted to cope with alternative assumptions or specifications regarding the handling costs, the influence of other merchandising variables, etc. For example, should the total assortment sales volume be sensitive to space allocation, the ϵ-elasticity could be taken into account through the substitution of (12) for (8) to determine the reallocation to be implemented in $t = 3$. It could be calibrated by comparing the level reached by aggregate sales in $t = 2$ (Q_2) to the initial one (Q_1). If we refer to (13), $Q_2/Q_1 = [\prod_{i=1}^{n} s_{i2}/(S/n)^n]^{\epsilon}$ and solving for ϵ, we get an acceptable estimate, $\hat{\epsilon} = [\ln Q_2 - \ln Q_1]/[\sum_i \ln s_{i2} - n \ln (S/n)]$. Observations gathered in period 3 could also be used to check its reliability.

Through the design proposed in Table 10, SH.A.R.P. parameters can be progressively refined and updated, while the performance of the space-allocation procedure can

TABLE 9

Contribution of SH.A.R.P. vs. Alternative Rules in the Cash-Battard Case

Percentage Increase in Profit Expected from the Implementation of SH.A.R.P. Final Iteration Result	
Over current actual profit	+9.25%
Over level expected from PROGALI	
—First iteration	+2.23%
—Steady State	+21.86%
Over level expected from the implementation of SH.A.R.P. 1st iteration result	+0.019%

TABLE 10

SH.A.R.P. Implementation Process

Period	Treatments	Measurement
$t = 0$	None: preliminary analysis of current allocation procedure. Data on space repartition (σ_{j0}), sales shares (m_{j0}) and relative contribution to profit (r_{j0}).	Prior guess of β^*, i.e. the ratio of the visibility impact to the relative incidence of handling costs. Alternatives: subjective estimate, or derivation of its implicit value from the application of (10) to data collected.
$t = 1$	Equi-allocation of available display space, $s_{j1} = S/n$.	Using the sales shares observed at the end of the 1st period (m_{j1}), estimate the preference coefficients through,[10] $\hat{\alpha}_j = m_{j1}$.
$t = 2$	Allocation guided by (9), $\sigma_{j2} = \hat{\beta}^* r_{j1} + (1 - \hat{\beta}^*) m_{j1}$, and $s_{j2} = \sigma_{j2} S$.	Estimate of the visibility impact parameter, $\hat{\beta} = \Sigma_i w_i [\ln m_{i2} - \ln m_{i1}] / \Sigma_j w_j^2$, with: $w_j = \ln s_{j2} - \frac{1}{n} \Sigma_i \ln s_{i2}$.
$t = 3$	Eventually revise the allocation according to the outcome of SH.A.R.P. heuristic, i.e. $\sigma_{j3} = \sigma_j^{(F)}$, where $\sigma_j^{(F)}$ denotes the final result to which (7) or (8) converges, using $\hat{\beta}$. Only if changes to be implemented are significant enough.	Control stability of the experimentation results by e.g., re-estimating β through a comparison of the observations collected in period 3 with those gathered in periods 1 and 2. Evaluate variations in profitability.

simultaneously be improved. Even in the worst cases, experimentation costs can thus be covered by the resulting profit increases.

6. Further Developments

In the CASH-BATTARD experiment, the prior guess of β^*—which reflects the management's implicit judgment about the relative magnitudes of the impact of item visibility and of their handling costs—led to a reallocation that resulted in a significant increase in the assortment profitability (+11.8%). This very fact strongly suggests that managerial experience could be more systematically called upon.

Along those lines, we are presently developing a P.C. interactive variant of SH.A.R.P. which integrates additional merchandising variables, such as shelf height, special displays, etc. Their impact can equally be modeled either through variations in the α_i-parameters, or through the introduction of coefficients serving as weighting factors of the space units.

SH.A.R.P. assumes fully symmetric patterns of substitution within the homogeneous product assortment considered. But because of either brand loyalty or package size purchasing habits, interactions may be stronger either between items of the same brand's line or between those of identical size. Bultez et al. (1988) currently study to what extent those asymmetries may affect space allocation.[11]

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Appendix A. On a Particular Extreme Case

Anderson and Amato (1974) recommend that each item, selected to be on display, is to receive *one* facing, except for the most profitable one to which all the remaining space available must be allocated. We show hereafter that such a peculiar norm can be traced back to the simplifying assumptions introduced in their model. Indeed they ignore replenishment costs and product-class sales elasticities, and further assume that $\alpha_j = \beta_j = 1$, or $m_j = \sigma_j$. With ϵ_j and C_j equal to zero for all j values, our own rule—i.e. (7)—reduces to $\sigma_j = \beta_j^*[(g_j m_j/\sum_c g_c m_c) - m_j]$ and letting $\bar{g} = \sum_c g_c m_c$, $\sigma_j = (\beta_j^* m_j/\bar{g})(g_j - \bar{g})$.

Applying the latter result would lead to an allocation whereby only the most profitable item would be displayed on the shelf, eliminating all the others. For an intuitive proof of such an extreme result, consider that at least one item's—let us say item k—gross margin is lower than, or equal to, the weighted average, i.e. $g_k \leq \bar{g}$. Then item k should be dropped from the product line. But once it has been eliminated, another item would evidently have a gross margin lower than the average, hence it would also have to be phased out. . . . Ultimately, only one item would be left and displayed, i.e. the most profitable one. That all other items are allowed one facing in the Anderson-Amato's case is due to the minimum visibility constraint imposed a priori for the items selected. One could argue that when only one item is left the average margin is equal to the highest gross margin, thus the corresponding space share would be zero should the allocation rule apply again. But this is a degenerate case, since shrinking the assortment down to one single item eliminates all space allocation problems within it.

Appendix B. Steady State Allocation Resulting Ultimately from a Consistent Application of the PROGALI Rule

PROGALI allocates the available space proportionally to the items' relative contributions to the assortment gross margin, i.e.

$$\sigma_i = g_i q_i / \sum_c g_c q_c = g_i m_i / \sum_c g_c m_c. \tag{B.1}$$

Starting from an equi-distribution of space, i.e. $s_i^{(0)} = S/n$, the initial sales shares are determined by $m_i^{(0)} = \alpha_i/\sum_c \alpha_c$ (see step # 0 in SH.A.R.P. heuristic), since $\beta_i = \beta$, for all i values. Therefore a first application of the PROGALI rule yields $\sigma_i^{(1)} = g_i\alpha_i/\sum_c g_c\alpha_c$.

This allocation leads to a new repartition of sales shares, $m_i^{(1)}$, which once substituted into (B.1), defines a new allocation, $\sigma_i^{(2)}$. Such a feedback-loop substitution process will ultimately converge to the steady state allocation, $\sigma_i^{(F)}$, which per definition should solve the following set of $(n - 1)$ independent equations,

$$\sigma_i^{(F)} = g_i m_i^{(F)} / \sum_c g_c m_c^{(F)}, \qquad \text{or}$$

$$\sigma_i^{(F)} = g_i \alpha_i [\sigma_i^{(F)}]^{\beta} / D^{(F)}, \tag{B.2}$$

with $D^{(F)} = \sum_c g_c \alpha_c [\sigma_c^{(F)}]^{\beta}$ and for all i values.

Dividing (B.2) by $[\sigma_i^{(F)}]^{\beta}$, we get $[\sigma_i^{(F)}]^{(1-\beta)} = g_i\alpha_i/D^{(F)}$ and solving for $\sigma_i^{(F)}$, $\sigma_i^{(F)} = [g_i\alpha_i/D^{(F)}]^{1/(1-\beta)}$. Since $\sum_i \sigma_i^{(F)} = 1$, we obtain

$$\sigma_i^{(F)} = [g_i\alpha_i]^{1/(1-\beta)} / \sum_c [g_c\alpha_c]^{1/(1-\beta)}. \tag{B.3}$$

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